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A MULTI-STAGE OPTIMIZATION-BASED APPROACH FOR THE LINER SHIPPING PROBLEM

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The central contribution of this paper is to provide a decision-support methodology for a broad class of inter-related problems in liner shipping. We aim at the optimization of liner networks by transforming the total network design into a sequential multi-stage optimization process in terms of *Ship Routing & Scheduling*, *Fleet Deployment*, and *Transshipment*. By fixing the various sources of non-linearity and by breaking down the total network design into the sequential solution of the aforesaid set of subproblems we have managed to accomplish our goals via the use of Linear, Dynamic and Integer Programming. The stages of the methodology are not completely autonomous; conversely, they interact in a *dynamic* way.

Keywords: maritime transportation; ship routing; fleet deployment; liner networks.

1. INTRODUCTION

Liner shipping is one of the two main components of ocean transportation, chartered shipping being the other one. Whereas chartered shipping operates in a perfectly competitive environment and mainly concerns bulk cargoes (liquid or dry), liner shipping operates

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typically in a cartelized environment and usually concerns unitized cargoes (containers and ro-ro cargoes). Liners maintain a regular schedule and their operations are inextricably linked with intermodal transportation.

The present paper aims to provide a decision-support optimization-based methodology for the *Liner Shipping Problem* (LSP). The LSP is prohibitively complex computationally to be practically formulated and solved in a single-pass way. Consequently, we have applied a generic multi-stage optimization-based methodology that breaks it down to more manageable subproblems. Thus, the LSP consists of the following components: (a) assigning a sequence of ports and times of each route to a vessel (*Ship Routing and Scheduling)*; (b) allocating the vessels in the fleet to specific trade routes (*Fleet Deployment)*; and, optionally, (c) *Transshipment* and optimization of regional sub-networks (*hub-and-spoke* vis-à-vis *direct-calls)*.

For a recent review of the routing and scheduling status and perspectives concentrated on ship operations one can refer to Christiansen et al. (2004). Various versions of the Fleet Deployment Problem (FDP) are presented in Jaramillo and Perakis (1991), and Powell and Perakis (1997). To our best knowledge and according to our taxonomy, no published work exists regarding the LSP.

2. METHODOLOGY- MATHEMATICAL FORMULATION

2.1. Methodology assumptions

Our principal assumptions are the following:

- *Speed of ships*: All ships assigned to the same route sail at the same speed to keep *frequency of service* constant. The most profitable speed is a priori established and is not a decision variable to our problem.
- *Resistance of Ships*: Once speed is assumed fixed, ship resistance is assumed to be known and independent of the loading conditions of the vessels.
- *Cargo Movements and Demand Forecasting*: The total amount of cargo to be dispatched per annum between pairs of ports is independent of the service frequency, demand being generated uniformly throughout the year.

2.2. Methodology Outline

One may refer to the simplified optimization process scheme shown in Figure 1. First, the model determines the *sequence of ports* in each route. Within this module, the methodology can choose different alternative models depending on whether or not demand is excessive. Second, the *frequency of service* is decided. Third, we perform *re-routing/minor routing*. Fourth, we apply an Integer Programming Model for the *allocation of ships to routes (fleet deployment)*. Finally, we can optionally use the *transshipment* module for the optimization of regional subnetworks. More details of this procedure are as follows.

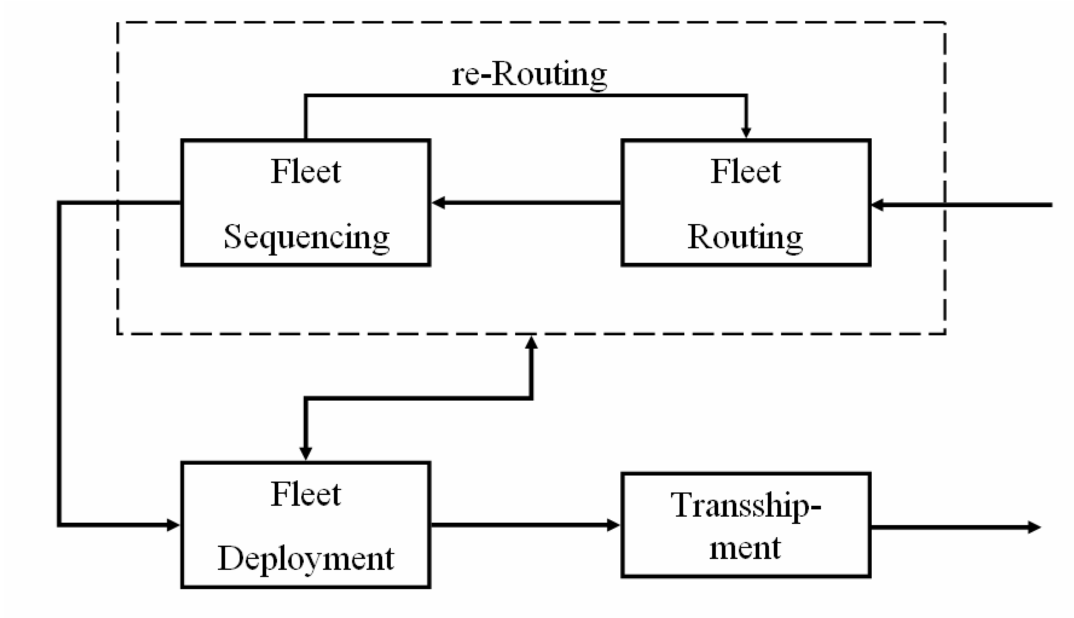

Figure 1. Simplified multi-stage optimization process scheme

2.3. Fleet routing

2.3.1. Initial sequencing

As regards routing determination, our methodology uses one of two different models depending on whether our fleet can satisfy all the demand or not.

a) Non-excessive demand:

For the case of non-excessive demand, our methodology assumes that the number of routes as well as the ports of each route are initially fixed. For the purpose of port sequencing determination, a dynamic programming computer program (TSPdyn) can be applied based upon the formulation of Held & Karp (1962): Starting from an initial and fixed set $S \subseteq \{2,3,\ldots,k\}$ and $k \in S$, we let C(S, k) be the optimal cost, i.e. distance, of starting from node 1, visiting all the nodes in S, and ending at node k.

We begin by the boundary conditions

$$C(\{k\},k)=d_{1k} \quad \text{for all } k=2,\ldots,n. \qquad (1)$$

To calculate C(S, k) for |S| >1, we can see that it obeys the following recursive relationship.

$$C(S,k)= \min_{m\in S-\{k\}} [C(S-\{k\},m)+d_{mk}] \qquad (2)$$

b) Excessive demand:

In most sequencing applications the model does not take into account instances where, for some reason, it is not possible to satisfy all the demand. In this case, several alternative formulations exist. These are the Profitable Tour Problem (PTP), the Orienteering Problem (OP) and the Prize-Collecting Traveling Salesman Problem (PCTSP). In the PTP the aim is to find a circuit that minimizes travel costs minus collected revenue. In the OP the travel cost objective is stated as a constraint and the aim is to find a circuit that maximizes collected revenue such that travel costs do not exceed a preset value $c_{\max}$. In the PCTSP, the revenue objective is stated as a constraint and the aim is to find a circuit that minimizes travel costs and whose collected profit is not smaller than a preset value $p_{\min}$. Their mathematical formulation is as follows:

A binary x_{ij} is associated to every arc $(\upsilon_i,\upsilon_j) \in A$, and is equal to 1 if and only if the corresponding arc is used in the solution, and a binary variable y_i is associated to every vertex $\upsilon_i \in V$, and is equal to 1 if and only if the corresponding vertex is visited. A revenue p_i is associated with each vertex and a cost c_{ij} with each arc.

The PTP, the OP and the PCTSP share a common set of constraints:

$$\sum_{v_j \in V \setminus \{v_i\}} x_{ij} = y_i \qquad (\upsilon_i \in V), \tag{3}$$

$$\sum_{v_i \in V \setminus \{v_j\}} x_{ij} = y_j \qquad (\upsilon_j \in V), \tag{4}$$

subtour elimination constraints (see below), (5)

$y_1 = 1$, (6)

$x_{ij} \in \{0,1\} \quad ((\upsilon_i,u_j) \in A)$, (7)

$y_i \in \{0,1\} \quad ((\upsilon_i,u_j) \in A)$. (8)

In the case of the PTP, the formulation is:

$$\text{Maximize} \quad -\left(\sum_{(v_i,v_j)\in V} c_{ij}\, x_{ij} + \sum_{v_i \in V} p_i\, y_i \right) \tag{9}$$

or alternatively: $$\text{Minimize} \sum_{(v_i,v_j)\in V} c_{ij}\, x_{ij} - \sum_{v_i \in V} p_i\, y_i, \tag{10}$$

subject to (3-8).

For the OP, the formulation is:

$$\text{Maximize} \sum_{v_i \in V} p_i\, y_i, \tag{11}$$

subject to (3-8), plus the additional constraint

$$\sum_{(v_i,v_j)\in V} c_{ij}\, x_{ij} \le c_{max} \tag{12}$$

For the PCTSP, the formulation is:

$$\text{Minimize} \sum_{(v_i,v_j)\in V} c_{ij}\, x_{ij}, \tag{13}$$

Subject to (3-8) plus the additional constraint

$$\sum_{v_i \in V} p_i\, y_i \ge p_{min}. \tag{14}$$

Regarding (5), we choose the following subtour elimination constraints:

$$u_i - u_j + Nx_{ij} \le N - 1 \text{ (for } i \ne j;\ i=2, 3, \ldots, N;\ j=2, 3, \ldots, N) \tag{15}$$

2.3.2. Other considerations in routing

It is highly possible that considerations such as precedence constraints or marketing factors may determine a sequence of ports in each route that is different from the optimal one as described by (1-2) above. We can compare the routes that are finally determined, if different from the output of TSPdyn, by means of the following formula:

$$ERD_r = \frac{d_r - d_{TSPr}}{d_{TSP_r}} \times 100\%\text{, for each } r = 1,\ldots,R \tag{16}$$

ERD_r is a numerical factor showing us how "profligate" in terms of sailing distance is the route r. ($ERD_r \ge 0$, $ERD_r = 0$ in the case that the final route is the output of TSPdyn)

d_r is the total sailing distance of (actual) route r

d_{TSP_r} is the total sailing distance of the TSP output route

If we want to acquire a quantitative sense of the distance "profligacy" in all R routes together, this can simply be the average of ERD_r:

$$\overline{ERD} = \frac{1}{R} \times \sum_{r=1}^{R} \frac{d_r - d_{TSPr}}{d_{TSP_r}} \times 100\% \tag{17}$$

2.4. Deciding Frequency of service and re-Routing

The formulation here is based on the work of Perakis and Jaramillo (1991).

2.4.1. Amount of cargo moved

Given a tri-dimensional matrix Q representing the amounts of cargo (TEUs) to be moved per year from port i to port j on route r, the amounts of cargo to be loaded or unloaded in every port are:

$$Q_{ir} = \sum_{j=1}^{I_r} [\, Q_{ijr} + Q_{jir} \,] \qquad (18)$$

Q_{ir} is the amount of cargo to be moved (loaded or unloaded) p.a., by all ships at port i on route r

Q_{ijr} is the amount of cargo to be carried p.a. from port i to port j on route r

Q_{jir} is the amount of cargo to be carried p.a. from port j to port i on route r

I_r is the number of ports on route r

The targeted number of voyages p.a. defines the amount of cargo that has to be loaded and unloaded per call (i.e. per voyage) at each port:

$$q_{ir} = Q_{ir}\, [F_r / 365] \qquad (19)$$

q_{ir} is the amount of cargo to be unloaded and loaded at the i-th port of route r.

F_r is the frequency of service.

2.4.2. Cargo levels on board

Thereinafter, a C++ computer model (titled Clev) computes the cargo levels, frequencies of service and minimum required capacities based on the following formulation:

$$L_{ijr} = \sum_{f=1}^{i} \sum_{g=j}^{f} Q_{fgr} \quad \text{(for i} = I_r \text{, where: } I_r \text{ is the last port in route r)} \qquad (20)$$

$$L_{ijr} = \sum_{f=j}^{I_r} \sum_{g=j}^{f} Q_{fgr} + \sum_{f=1}^{i} \sum_{g=1}^{f} Q_{fgr} + \sum_{f=1}^{i} \sum_{g=j}^{I_r} Q_{fgr} \qquad \text{(for i} \neq I_r\text{)} \qquad (21)$$

$$L_r = \max L_{ijr} \qquad (22)$$

L_{ijr} is the amount of cargo on board for a ship sailing from port i to port j on route r, for the case of one voyage per year

L_r is the amount of cargo in the most heavily loaded leg

Q_{fgr} is the amount of cargo to be carried per year from port f to port g on route r

Then, the minimum required capacity of ships that are to operate on route r is:

$$RC_r = L_r / (365 / F_r) \qquad (23)$$

where F_r is the established frequency of service.

If ships of type k with given capacity V_k (in TEUs) are assigned to route r, the minimum required number of voyages per year in that route is:

$$RV_r = L_r / V_k \qquad (24)$$

and the corresponding value of frequency of service is:

$$F_r = 365 / RV_r \qquad (25)$$

2.4.3. Tools for fleet re-routing and network efficiency evaluation

We perform *re-routing/minor routing* through visualizing network efficiency by certain graphs. In particular, graphs of (23) show the frequency of service vs. capacity tradeoff in the different routes (RC_r versus frequency of service, F_r). A graphical representation of (20), (21) show the loading condition of the ships in the various legs of a specific route, offer an insight to the utilization of the ships and provide hints for minor routing or frequency of service modifications. At this point, we can identify ship-route incompatibilities due to inadequate cargo capacity. The fixing of the frequency of service is required in order to avoid non-linearity afterwards.

Suppose we have pre-allocated certain K ships to certain routes. The initial utilization of each ship operating on a certain route can be further examined via the two following formulas:

$$\overline{ASU}_{(k,r)} = \frac{\sum_{i=1}^{I_r} L_{i(i+1)r}}{I_r \times V_k} \quad \text{(note: for i=} I_r \text{, } L_{i(i+1)r} \text{ returns } L_{(Ir)1r}\text{)} \qquad (26)$$

where $\overline{ASU}$ is the simplified Average Ship Utilization, and

$$\overline{\overline{ASU}}_{(k,r)} = \frac{\sum_{i=1}^{I_r} L_{i(i+1)r} * d_{i(i+1)r}}{\sum_{i=1}^{I_r} d_{i(i+1)r} \times V_k} \qquad (27)$$

where: $\overline{\overline{ASU}}_{(k,r)}$ is the leveled Average Ship Utilization,

$\sum_{i=1}^{I_r} d_{i(i+1)r}$ is the total sailing distance of route r;

$d_{i(i+1)r}$ is the distance between two consecutive ports i, i+1 in route r.

Now the utilization of the fleet can be examined via the following formulas:

$$AFU = \frac{1}{K} * \sum_{k=1}^{K} \overline{ASU}_{(k,r)} \qquad (28)$$

$$\overline{AFU} = \frac{1}{K} * \sum_{k=1}^{K} \overline{\overline{ASU}}_{(k,r)} \qquad (29)$$

$$\overline{\overline{AFU}} = \sum_{k=1}^{K} (\overline{\overline{AFU}}_{(k,r)} \times V_k) \times \frac{1}{\sum_{k=1}^{K} V_k} \qquad (30)$$

AFU is the simplified Average Fleet Utilization

$\overline{\overline{AFU}}$ is the single-stage leveled Average Fleet Utilization

$\overline{\overline{AFU}}$ is the two-stage leveled Average Fleet Utilization.

Our understanding is that from a vehicle routing perspective it would be an omission not to take into account the effects of specific route characteristics on a certain ship's cost-size evaluation. The following formula reflects the rather convoluted relationship among ship size, operating route and cost. $ES_{kr} = \frac{C_{kra}}{V_k}$ (31)

ES_{kr} is the economies of scale factor for a k type ship operating on route r ($/TEU)

C_{kra} are the (total) annual operating costs of a type k ship in route r

At this point the decision-maker should reevaluate the routes and the sequence of ports in each one. The routing & scheduling of the fleet must be finalized before embarking on the next optimization stage.

2.5. Allocation of Ships to Routes

To allocate ships to routes, the following Pure Integer Programming model is proposed (Powell and Perakis, 1997). The objective function in the model minimizes the sum of the operating costs and the lay-up costs.

$$\text{Minimize} \sum_{k=1}^{K} \sum_{r=1}^{R} C_{kra} N_{kr} + \sum_{k=1}^{K} Y_k e_k \qquad (32)$$

where e_k is the daily lay-up cost for a type k ship

and the decision variables are:

N_{kr} number of type k ships operating on route r

Y_k number lay-up days per year of a type k ship

Subject to: $$\sum_{r=1}^{R} N_{kr} \leq N_k^{\max} \quad \text{for each type k ship} \qquad (33)$$

where: $N_k^{\max}$ maximum number of type k ships available

$$\sum_{k=1}^{K} t_{kra} N_{kr} \geq M_r \quad \text{for all r} \qquad (34)$$

$$t_{kra} = T_k / t_{kr} \qquad (35)$$

t_{kr} voyage time of type k ship on route r

t_{kra} yearly voyages of a type k ship on route r

T_k shipping season for a type k ship

M_r number of voyages required per year on route r

$$N_{kr} = 0 \text{ for specified (k,r) pairs} \qquad (36)$$

$$Y_k = 365\, N_k^{\max} - T_k \sum_{r=1}^{R} N_{kr} \qquad (37)$$

(33) are the ship availability constraints; (34) ensure the minimum required frequencies of service are met; (36) are the ship-route incompatibility constraints; finally, (37) are the lay-up time (includes dry-docking and repair time) constraints.

2.7. Transshipment

Once the above stages have been successfully applied, we have an efficient network from the routing and scheduling point of view as well as an optimal one from the fleet deployment point of view. At this point, we will have to discuss if a transshipment evaluation is necessitated. Its ambition will be to serve the special needs or inefficiencies of regional sub-networks. The planning problem consists of choosing which of a possible set of predefined routes to use. We think that a simulation model works very well for this purpose insofar as simulation models are similar to gaming models except that all human decision-makers are removed from the modeling process. Mathematical formulations of the transshipment module will not be presented herein.

2.8. Computational Results

Given that one of the subproblems of the LSP, the TSP, is NP-hard, the same must be true for LSP itself. PTP, OP, and PCTSP are also NP-hard as a TSP instance can be stated as a PTP, OP, or PCTSP instance by defining arbitrarily large profits on vertices. With the aforesaid dynamic programming formulation, we need memory equal to $O(n2^n)$ locations and CPU time equal to $O(n^2 2^n)$. But this is not a problem since the number of nodes are not anticipated to be that high. For 18-node instances the computational time was 28, 27, and 21 seconds for the PTP, OP, and PCTSP (respectively) on an Intel Pentium M processor 1.8 GHz with 512MB of RAM using a branch-and-bound algorithm. For the Fleet Deployment problem and with the same hardware, we had 59 integer variables and the elapsed run time was 1 second using a branch-and-bound algorithm.

In the OP (PCTSP) the optimal solutions have cost (profit) outputs which were very near to their upper (lower) bounds, correspondingly. In addition, in a certain scenario for the PTP, we increased all the travel costs by 30% each (indicating the continuous rise in fuel oil). Perhaps surprisingly, the output had a lower total cost. This occurred since our objective criterion is the profit (revenues minus costs) so the model chose to visit much less nodes to achieve maximum profit. More details on the results of a FDP realistic application are in Tsilingiris (2005) but are not presented here due to space considerations.

3. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

The liner-shipping problem (LSP) is prohibitively complex computationally to be practically formulated as a single-stage process. Consequently, we have presented and applied a generic multi-stage optimization-based methodology for it. The contribution of the present work is highlighted by the facts that (to our best knowledge) no former similar published work exists and that the number of papers using exact techniques to systematically analyze liner fleets lags far behind the swift increase in the capacity of the global liner business.

We feel that future research in the *liner-shipping problem* or its strategic "counterpart", the *liner-network design problem*, can be extremely rewarding and, probably, abreast of future ocean-market-driven consulting services. The growth of containerization makes the need for sophisticated *intermodal*-network design very clear. Liner shipping has a crucial function in the integration of waterborne transport into a multi-modal door-to-door supply chain. We can utilize the theoretical work on TSP, Artificial Intelligence techniques and the rapidly decreasing computational costs to create expert robust systems in liner shipping which would have available algorithms and select an appropriate for each particular application.

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