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A synthesis algorithm for an oil spill problem of complementary locations on networks

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Abstract

This paper develops a synthesis algorithm on networks for the problem of (a) locating appropriate levels and types of cleanup capability to respond to oil spills that may occur in a given area, and (b) allocating such capability among points of high oil spill potential in that area. The heuristic algorithm takes into account frequency of spill occurrence, variability of spill volumes, differing cleanup technologies, equipment efficiency and operability, fixed costs of opening facilities, equipment acquisition, transportation and operating costs, and costs of damage as functions of spill volume and level of response. The algorithm can also accept stipulations on response times. The results of the heuristic algorithm have been compared with results obtained by a Linear Programming (LP) formulation, and the highest deviation observed was around 1%. The advantage of the developed algorithm is apparent in real-life oil spill problems which are large-scale in nature and which cannot be solved by the LP procedure. © 2001 Published by Elsevier Science Inc. All rights reserved.

1. Introduction

1.1. Background

Oil spill response is interpreted as the emergency action that must be taken to mitigate damage that may be caused once an oil spill occurs. Governments, industry and society in general are concerned about massive spills, as well as the smaller spills that occur on a day-to-day basis. Part of the emergency action to mitigate damage concerns the dispatching of specialized cleanup equipment to the site of the spill in order to contain, recover or disperse the spilled oil.

This paper concerns the problem of deciding where to locate adequate capability to respond to oil spills that may occur in a given area. In addition to considerations of location, this problem generally calls for decisions concerning the proper levels and types of equipment to be stockpiled,

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as well as for policies regarding the allocation of such capability among points or zones of high oil spill potential. The purpose of the paper is to formulate a model for the oil spill response problem, outline an approach to the solution of this problem using a synthesis algorithm on networks, and compare and discuss its results and those of a Linear Programming (LP) approach [6] to the problem. Oil spill response decisions typically involve planning horizons of considerable duration (e.g., 5–15 years). Since those decisions have to be made before actual spill incidents in the area of interest occur, the planner has to base those decisions on, among other things, probabilistic information on the number and volume of such spills, as well as on assessment of the potential consequences of any particular spill event under a prescribed response.

Recognizing the connection between oil spill response decisions and operational decisions, this paper focuses on strategic decisions. However, since such a connection exists, and for the sake of making this paper self-contained, simplifying assumptions are included regarding spill response operations, which are represented in a fairly aggregate (but realistic) fashion. It is fair to say that the topic of oil spills has to date received far less attention in the OR/MS literature than it really deserves. Nonetheless, several studies are worthy of note: [1] developed a “goal interval programming” model to aid resource allocation decisions in the US Coast Guard’s (USCG) Marine Environmental Protection program. One of the uses of that model related to pollution response. [2] developed a “chance constrained goal programming” model to aid USCG managers in formulating policies with respect to planning for various types of equipment required to control major spills. A Transportation Systems Center study [8] addressed the question of how much equipment the USCG should stockpile in order to satisfy the Presidential directive set forth by Jimmy Carter in his message to Congress in March of 1977: “The goal is to respond adequately to a 100,000-ton spill within 6 h”. However, none of the above or other studies really provided an integrated framework in which oil spill response decisions at all levels could be systematically analyzed and tradeoffs explicitly evaluated. Consequently, Psaraftis et al. [7] attempted to provide the appropriate research framework, and investigated the structure of the problem. They formulated it as a Mixed Integer Programming Problem, illustrated by an application of the model in the New England region of the US This paper is, in fact, an extension of this work.

1.2. Type of problem and objectives

It is worth mentioning that experience shows that it is not at all unlikely that the response to a particular spill will come from more than one facility. This is particularly true for very large, catastrophic spills, where the response typically originates from two or more mutually supportive (or “complementary”) locations because each facility is rarely able to handle such spills by itself. Note that this contrasts with the usual assumption in more “classical” facility location problems (K-median, K-center, etc.), where each demand point is served by only one facility (typically its “closest” median or center). In strategic planning for oil spills, therefore, one should pay attention to the complementary nature of the response.

A natural question in designing oil spill response systems is what is the objective function to be optimized. This paper assumes that the objective is to minimize the expected total of response system costs and costs due to damage from spills that may occur in the area, the latter costs being open to a user-specified weighting. Taking into consideration both system costs and damage costs as part of the problem objective makes sense intuitively, because any response system requires funds, and one would like to know not only how much a system would cost but also how much damage that system would avert. However, it is clear that such an objective function implies not only risk neutrality on the part of the decision-maker, but, equally importantly, that oil spill damage costs can be evaluated with some confidence. Risk neutrality is assumed both for analytical convenience and because very little or nothing has been reported to date regarding the risk preference structure of "society" regarding oil spills, and, for reasons similar to those discussed earlier regarding expected volumes, constraints are introduced that are able to take into account the decision-maker's aversion to risk, particularly when it comes to very large, very rare, catastrophic spills. Regarding the evaluation of damage, despite the general consensus that oil spill cleanup and damage assessment are not, and are never likely to be, precise arts [9], some progress has been made in this area in recent years, and related efforts are continuing.

This paper takes advantage of the work done by Psaraftis et al. [7], the purpose of which was to quantitatively evaluate the damage costs of an oil spill under a variety of scenarios. Such an approach takes as input spill-specific information (location, size, sea state, wind, oil type), area-specific information (inventory of environmental and economic resources) and information on the response, and produces estimates of damage, broken down into several categories (value of lost oil, organisms, beaches, marshes, recreation, etc.). This paper assumes that damage can be predicted as a function of several spill parameters and of the response to a spill, and describes how such information can be used in a decision-making process. Regarding damage weighting, its role is twofold: First, it can be used to represent how much the decision-maker is willing to pay in system costs in order to reduce damage costs by $1 (and, in that respect, a high value of that weighting increases the relative importance of damage costs vis-a-vis system costs). Second, the weighting can be used to perform sensitivity analysis on the evaluation of damage, which, as mentioned before, is never likely to be precise.

Finally, the approach to solving the oil spill problem employed is a synthesis algorithmic one, using the concept of complementary locations on networks. This approach and some of its components are described in [5,6] and [3].

2. The problem as network synthesis

The problem described requires the determination of equipment levels (cleanup capability) at various locations to respond (economically) to spills occurring at designated demand (risk) points. The spills are of a given magnitude expressed in terms of spill volume. A given allocation of cleanup capability means that a location i with proper cleanup equipment responds to a spill at a risk point j if it is economically justifiable to do so. We can define two sets of nodes: NI (location points) and NJ (demand points); and in addition a dummy node O. Let the unit cost of responding to a spill at j from location i be designated as c_{ij}. The cost of c_{ij} will be the sum of the costs of transporting equipment, cleaning up the spill, and the damage resulting from the delay incurred by the response having arrived from location i, multiplied, on an anticipated value basis, by the expected frequency with which spills occur at j. Once the strategic decision on equipment levels is made, the system is committed to a certain response level $q(i,j)$ at a per unit cost of c_{ij}. Similarly, once the strategic decision is made, the system is committed to the per annum acquisition cost of the equipment at location i. This cost may be expressed as $c'_{Oi}Q(O,i)$ where c'_{Oi} is the per unit acquisition cost and $Q(O,i)$ is the level of equipment acquired for location i. If given the strategic decision some of the demand at spill site j cannot (economically) be fully satisfied, then the system will incur the unsatisfied demand cost c_{Oj} per unit of demand that remains unsatisfied. Let this cost be $c_{Oj}q(O,j)$, where c_{Oj} is the per unit cost, and $q(O,j)$ is the demand at j that could not be satisfied. We are now in a position to describe this as a network synthesis problem for which a heuristic procedure has been developed.

The total demand at a risk point j is designated p_j, and it is clear that this demand is either satisfied by the response of $q(i,j)$ from some location i, or unsatisfied by a response level of

$q(O,j)$. The demand p_j at j can be described as a requirement between the risk point j and the dummy node O. This requirement can be satisfied either through a location i, in which case, the demand is fully satisfied, or directly from node O, in which case, the demand is fully unsatisfied. Clearly, one can also have a combination whereby the demand is partially satisfied. Also, if $c_{Oj} \leqslant c_{ij}$ for all $i \in \mathrm{NI}$, the solution will permit the entire demand to go unsatisfied. Fig. 1 depicts a situation in which there are two risk points j and j'. A given location i can serve *both* j and j'. However, the clean-up capability $Q(O,i)$ at i need not be the sum of $q(i,j) + q(i,j')$. This is based on the assumption that two or more spills can never occur simultaneously. It also implies that the capability $Q(O,i)$ should be as large as the maximum between $q(i,j)$ and $q(i,j')$.

Overall, the problem can be described as choosing "capabilities" on the arcs of the network shown in Fig. 1, which result in a minimum total cost, while supplying the levels p_j of demand for all $j \in \mathrm{NJ}$. Those capabilities to be located on the arcs (O,j), $j \in \mathrm{NJ}$ are the unsatisfied demand. Also allocation of capabilities to (O,i) means acquiring capability for location i.

The fixed costs of establishing a facility are not taken into account within the network synthesis approach, which assumes that a given facility can be established free of cost. The only expense incurred is the variable costs associated with the level of capability acquired. In order to tackle the fixed cost problem, the most obvious solution is to examine all possible combinations of facilities, and add on to the solutions obtained, the fixed costs of the facilities included in the combination. This would be extremely cumbersome, since for a problem with 20 possible locations S_{20} different problems need to be solved where S_n is given by

$$S_n = \sum_{r=1}^{n} \frac{n!}{r!(n-r)!} = 2^n - 1.$$

That is, even for moderate-sized problems $S - S_n$ is a very large number ($S_{20} = 1{,}048{,}575$). Note that it is possible to use for this problem an LP formulation with the methods of Mixed Integer

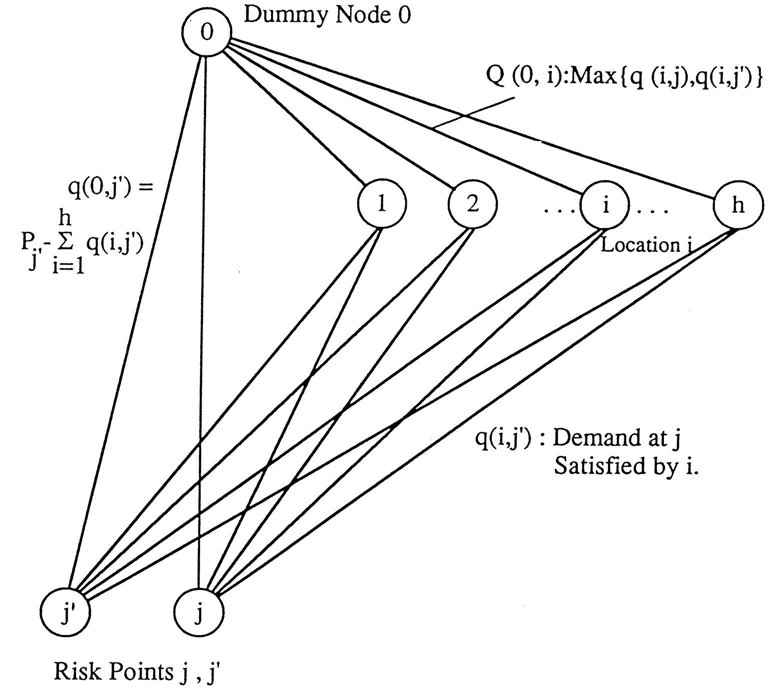


Fig. 1. Network synthesis, representation with two risk points.

Programming such as Branch and Bound or Lagrangean Relation. In Section 6 of this paper an Implicit Enumeration scheme is developed, which may be used either with the LP formulation to yield an exact solution, or with the heuristic procedure to obtain an approximate solution to the problem including fixed costs.

3. A heuristic procedure ignoring fixed costs

The heuristic procedure developed for solving the strategic problem is based on the network synthesis representation and relies on the following observations. The capability allocated to a location *i* should be no greater than the largest response emanating from location *i*. The responses from a location *i*, which equal the capability of that location may be thought of as marginal capacities, in the sense that a reduction in these responses would allow the reduction of the capability located at *i*. On the other hand, responses which are below the capability at location *i* may be thought to have surplus capacity in that they could be increased without increasing the capability at location *i*. These observations give rise to the twin concepts of Marginal Capacity Subgraphs and Surplus Capacity Subgraphs. The heuristic procedure uses these subgraphs to proceed from a feasible solution to an improved solution and terminates when no further improvement is possible. A counter-example is also presented later and indicates that the procedure need not terminate in an optimal solution.

We may define an additional terminology to be used in presenting the algorithm. The graph $G(N,A)$ is made up of two bipartite sets of nodes NI and NJ and a dummy node (see Fig. 2). The set of nodes NI are referred as the location set, and to the set of nodes NJ as the demand set. A dummy node 'O' is connected to all nodes in NI and in NJ. The arcs of the graph can be characterized under three groups. The capability arcs (O,NI) connecting the dummy node O to the nodes in NI, have the cost c'_{Oi} for each $i \in \mathrm{NI}$ of acquiring capability. The response arcs (NI, NJ) connecting nodes in NI to those in NJ, have the cost $c_{ij}, i \in \mathrm{NI}, j \in \mathrm{NJ}$ of responding to spills (demand), which comprise the transport, clean-up, and delay costs. The penalty arcs (O,NJ)

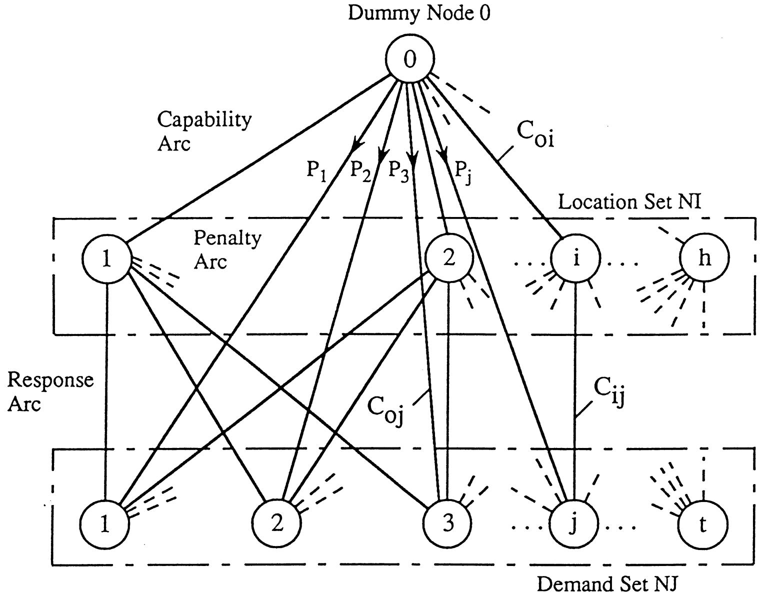


Fig. 2. The graph $G\{N,A\}$ with all the components involved in the network synthesis algorithm.

connecting the dummy node O to the nodes in NJ have the penalty cost $c_{Oi}, j \in$ NJ for incomplete response (damages due to unsatisfied demand). Also, a requirement p_j is specified between the dummy node 'O' and the nodes $j \in$ NJ. The problem is to find the least-cost allocation of capabilities to the arcs of $G = \{N, A\}$ which satisfy the requirements and minimize the total cost. Fig. 3 demonstrates an example problem.

The first step of the algorithm finds a feasible solution, which is also an upper bound on the total capability required on the capability arcs to satisfy the requirements p_j. This step does not allow any unsatisfied demand. The procedure finds the least-cost paths from O to j via some i^* and assigns capability $q(i^*, j) = p_j$. The value of the $Q(O, i)$ are obtained by finding the maximum response emanating from i.

Step 0: Find the least-cost path from node O to each $j \in$ NJ through some $i \in$ NI. Let $i^*(j)$ be the node $i \in$ NI for which $c'_{Oi} + c_{ij}$ is minimum, or find

$$c'_{Oi^*} + c_{i^*j} = \min_{i \in \mathrm{NI}} \left(c'_{Oi} + c_{ij} \right) \quad \forall j \in \mathrm{NJ},$$

let,

$$q(i,j) = \begin{cases} p_j & \text{if } i = i^*(j) \quad \forall j \in \mathrm{NJ}, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

$$Q(O,i) = \operatorname*{Max}_{j \in \mathrm{NJ}} q(i,j). \tag{2}$$

Designate F^0 as the Capacity Subgraph consisting of the arcs $(O, i^*(j)), (i^*(j), j)$ for all $j \in$ NJ.

The capacity subgraph F^0 represents a feasible solution to the problem, and the sum of the capacities $Q(O, i)$ is an upper bound on total capability since as much capability as necessary is located at the most advantageous location for each node in the demand set NJ. The initial capacity subgraph $F^0 = \{N, A_F^0\}$ for the problem of Fig. 3 is illustrated in Fig. 4. Then the marginal capacity subgraph $M^0 = \{N, A_M^0\}$ is created as well as the surplus capacity subgraph $S^0 = \{N, A_s^0\}$. For F^0, M^0 and S^0 the set of nodes remains the same and only the set of arcs is changed for the initial solution.

Include in M^0 all arcs $(O, i) \in A_F^0$ and all arcs $(i, j) = \{(i, j) : (i, j) \in A_F^0$ and $q(i_{ij}) = Q(ai)\}$. Label each (O, i) arc, $i \in A_F^0$, with

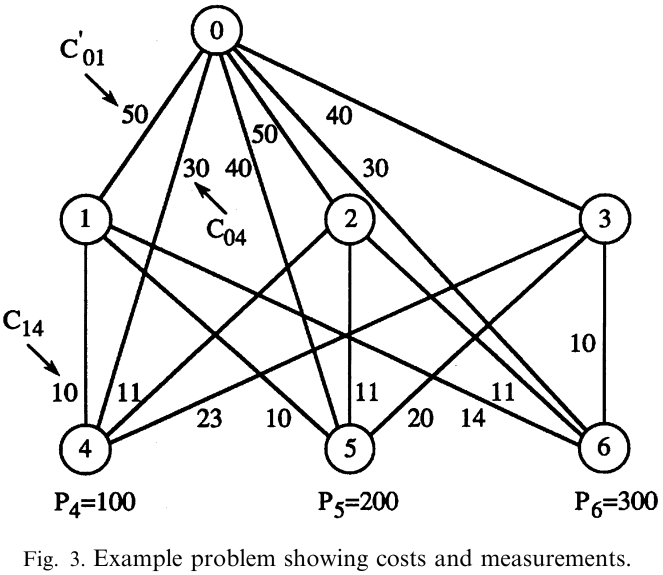


Fig. 3. Example problem showing costs and measurements.

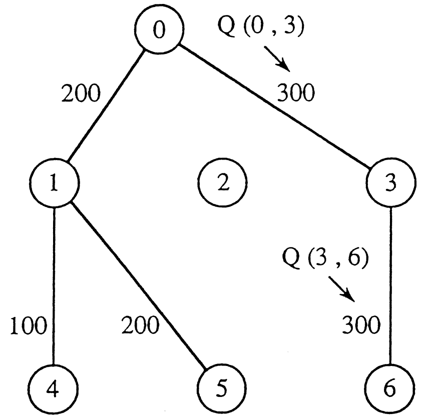


Fig. 4. Initial capacity subgraph F^0 showing arc capacities.

$$\delta_i = Q(O,i) - \underset{(i,j)\in(A-A^0_M)}{\mathrm{Max}} q(i,j) \quad \forall i \in N, \tag{3}$$

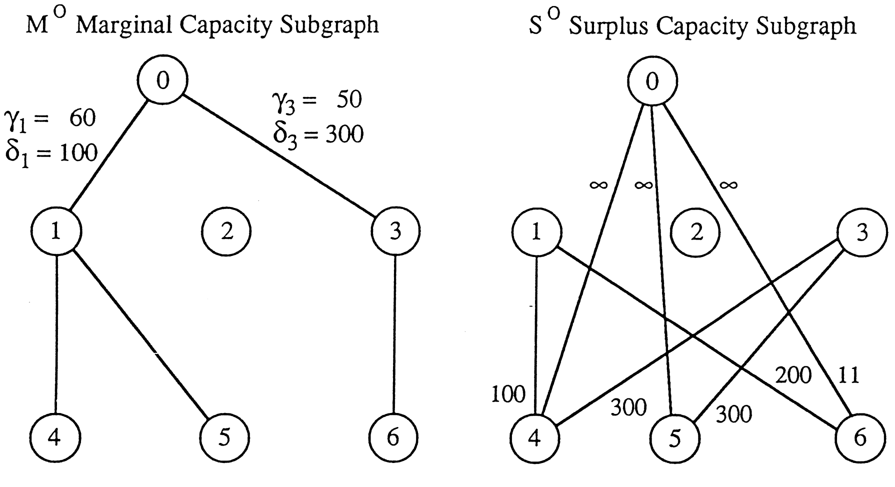
$$\gamma_i = c'_{Oi} + \sum_{(i,j)\in A^0_M} c_{ij} \quad \forall i \in N. \tag{4}$$

Include in S^0 all arcs $(k,j) \in A^0_M$, where, A_M is defined as the complement set of $A^0_M, k \in N$. Label each arc $(i,j) \in A$ with its surplus capacity $s(k,j)$ according to

$$s(k,j) = \begin{cases} Q(O,i) - q(k,j), & k \neq 0, \\ \infty, & k = 0. \end{cases} \tag{5}$$

Fig. 5 shows S^0 and M^0 for the example problems. The algorithm does not return to Step 0 at any future iteration. Step 0 provides an initial solution which is improved upon by Steps 1 and 2, until termination occurs in Step 3 when no further improvements are obtained.

Step 1 attempts to improve the objective by taking advantage of the surplus capacities available and thereby reducing the initial capability allocated at various locations *i*. Step 1 never considers the possibility of increasing the capability at any location as a means of reducing the overall objective.


M^O Marginal Capacity Subgraph

S^O Surplus Capacity Subgraph

Fig. 5. Surplus and marginal capacity subgraphs.

Following the initialization let $O = m$, where $(M^m) = \{N, A_M^m\}$ and S^m $\{N, A_s^m\}$.

Find α_j (before Step 1) to be associated with k^* according to

$$\alpha_j = \underset{k|(k,j)\in A_s^m}{\text{Min}} c_{kj} = c_{k^*j} \tag{6}$$

and set, $\eta_j = k^*$. Do this for all $j \in \text{NJ}$. Set QTERM = 0. Find i^* that maximizes Π in Eq. (7). That is,

$$\Pi = \underset{i\in\text{NI}}{\text{Max}} \left\{ \sum_{i|(i,j)\varepsilon A_M^m} (c_{ij} - \alpha_j) + c'_{Oi} \right\}. \tag{7}$$

If $\Pi < O$ go to Step 2.

Set QTERM = 1 and create δF^m as follows:

$$\sigma = \underset{i|(i,j)\in A_M^m}{\text{Min}} \{s(n_j), \delta_{i^*}\}, \tag{8}$$

$$Q(O, i^*) = -\sigma. \tag{9}$$

Define $J = \{j \mid (i^*, j) \in A_M^m\}$

$$\delta q(i^*, j) = -\sigma \quad \forall j \in \text{NJ}, \tag{10}$$

$$\delta q(n_j, j) = +\sigma \quad \forall j \in \text{NJ}. \tag{11}$$

Go to Step 3.

Fig. 6 shows the first iteration of Step 1.

In Step 1 the algorithm reduces the capability at a location i for which Π is positive; if there are more than one location satisfying the criterion ($\Pi > 0$) it picks the one that gives the most positive Π. This will clearly improve the solution since the summation in Eq. (7) is the net benefit in response costs resulting from a unit reduction in capability, and the second term c'_{Oj}, is the savings in acquisition costs. In the event that Step 1 fails to produce an improved solution the algorithm proceeds to Step 2.

In Step 2 the algorithm attempts to improve the solution by increasing the capability at some location. Improvements through the increase in capability are available primarily because of the possibility of consolidating capabilities and thereby saving acquisition costs. This will allow for example, allocation of capability at a central location which is not the closest to any one demand point and yet it is advantageous because it can serve a large number of demand points economically. Step 2 first identifies a set k^+ of candidate locations whose capability may be reduced given an increase in capability elsewhere in the system. A location i will exist in the set k^+ only if it is better to reduce the capability at that location rather than do "something else". The "something else" is explained in Step 2 in a mathematical manner.

Step 2: For each $j \in \text{NJ}$ determine $\beta_j = C_i^* j$ according to

$$\beta_j = \underset{i\in\text{NI}}{\text{Max}} \; c_{ij}. \tag{12}$$

Determine the set k^+ as follows:

$$k^+ = \left\{ k : \sum_{j|(k,j)\in A_M^m} \beta_j < \gamma_k \right\}, \quad k \in \text{NI} \tag{13}$$

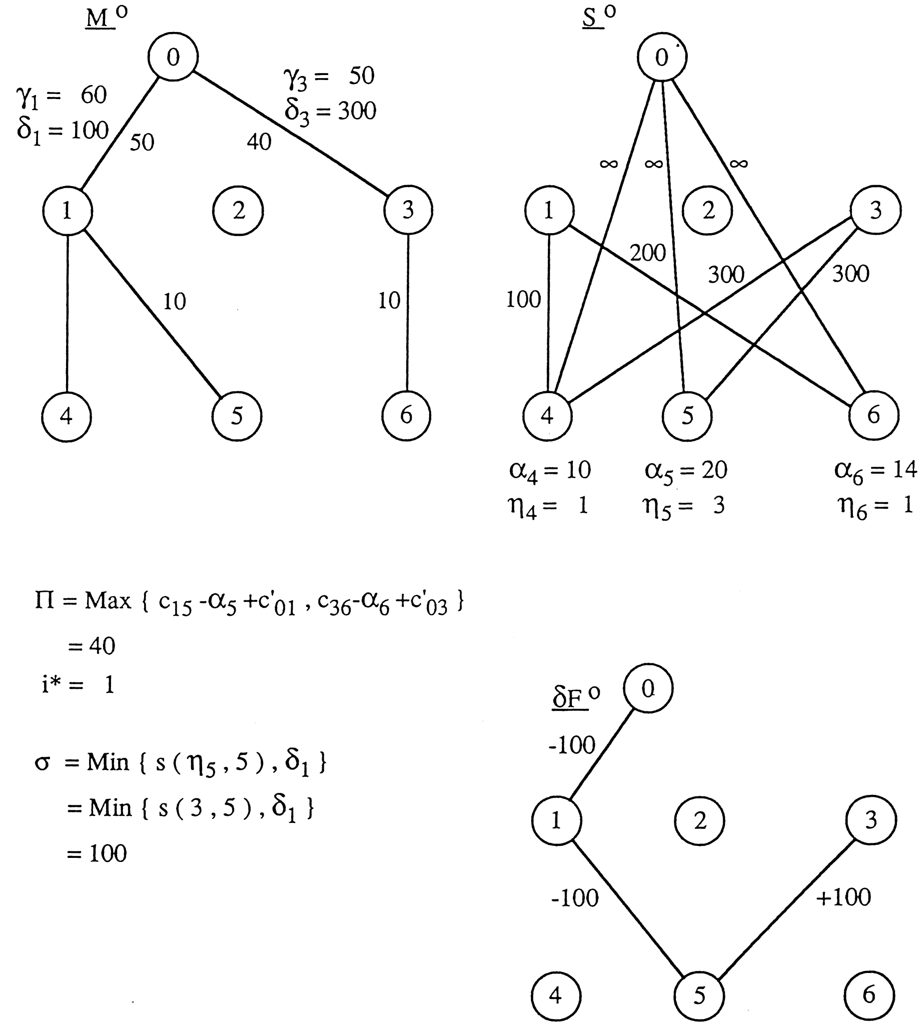


Fig. 6. First iteration of Step 1 of the algorithm for the example problem.

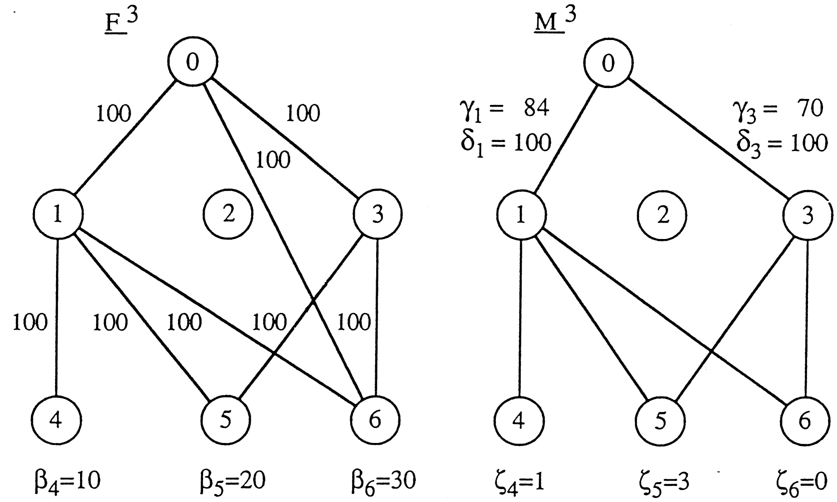
Fig. 7 illustrates the setting up of k^+. An increment in capability in the system could have replaced the most expensive response to demand j with a per unit cost of β_j, but then it could not have reduced $k \in k^+$, where

$$(k,j) \in A_M^m.$$

That $k \in k^+$ ensures that it is preferable to reduce k rather than to replace β_j. Step 2 next finds for each $i \in$ NI the set $k_i^+ \subseteq k^+$ for which an increase at the specific location i will allow a reduction in capability at $k \in k_i^+$.

For each $i \in$ NI determine k_i^+ as follows:

$$k_i^+ = \left\{ k : k \in k^+, k \neq i, \left[\gamma_k - \sum_{j|(k,j) \in A_M^m} \operatorname{Min}(c_{ij}, \alpha_j) \right] > 0 \right\}. \tag{14}$$



k = 1: $\beta_4 + \beta_5 + \beta_6 = 60 < \gamma_1 = 84$

k = 3: $\beta_5 + \beta_6 = 50 < \gamma_3 = 70$

$k^+ = \{ 1 , 3 \}$

Fig. 7. Determination of the set k^+ in Step 2 of the algorithm.

Reduce k_i^+ as follows:

Define $\bar{k}_j$ for $j \in$ NJ as

$$k_j^- = \left\{k : (k,j) \in A_M^m, k \in k_1^+\right\}. \tag{15}$$

Choose $k^* \in k_j^-$ that maximizes the following expression:

$$\rho = \mathop{\mathrm{Max}}_{k \in k\bar{j}} \left\{ \gamma_k - \sum_{j|(k,j)\in A_M^m} c_{ij} \right\}. \tag{16}$$

Let

$$k_i^+ := k_i^+ - k_j^- + k^*.$$

The set k_i^+ formed initially is later reduced to prevent conflicts. Such conflicts arise between elements of k_i^+ because they both serve the same demand point j. The set k_j^- are the elements of k_i^+ which are in conflict at demand point j. In the event of a conflict the algorithm retains the most advantageous element of k_j^- and removes the rest from k_i^+. The algorithm next identifies the subset $t_i^+ \in$ NJ. The t_i^+ is a set of locations j which exist in the Marginal Capacity Subgraph but for which no arcs (k_i^+, t_i^+) exist in the subgraph M^m. The algorithm proceeds as follows:

Establish the set $t_i^+ \in$ NJ where

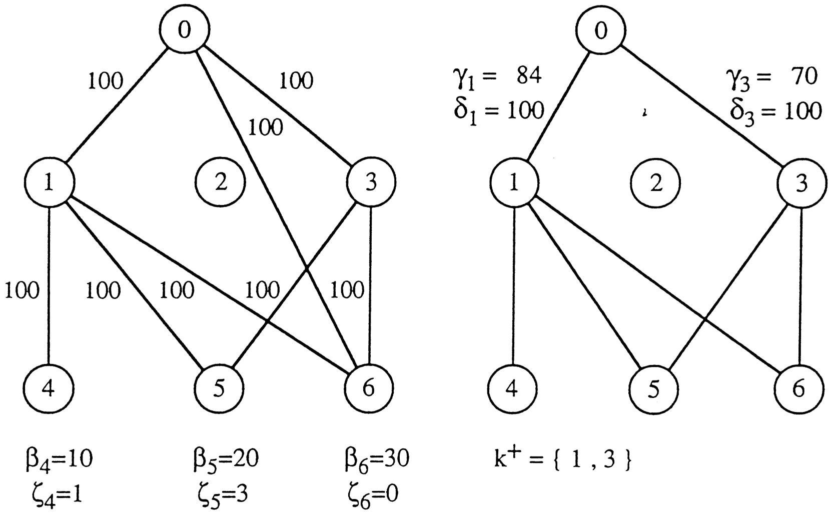
$$t_i^+ = \left\{t : (k_i^+, t) \in A_M^m, (\beta_t - c_{it}) > 0\right\}. \tag{17}$$

Calculate Π_i according to

$$\Pi_i = \sum_{k\in k_i^+}\left\{\gamma_k - \sum_{j|(k,j)\in A_M^m} \mathrm{Min}(c_{ij},\alpha_j)\right\} + \sum_{t\in t_i^+}(\beta_t - c_{it}) - c'_{Oi}. \tag{18}$$

The set t_i^+ therefore includes those demand points $t \in \mathrm{NJ}$ for which the new location I provides a net benefit, as expressed by the condition $(\beta_t - c_{it}) > 0$. If c_{it} is less expensive than β_t (most expensive current response to t) then establishment of the additional capability at i will benefit the objective at t. This benefit of course is conditional on $\left(k_i^+, t\right)$ not existing in M^m, since if it exists in M^m, then the additional capability at i would be utilized in the reduction at k_i^+. This, however, is the best improvement, owing to the observation (through the determination of k^+) that it is preferable to reduce the locations in k_i^+, rather than to substitute for the most expensive responses β_j.

Fig. 8 illustrates the derivation of k_i^+, t_i^+ and Π_i. The algorithm now proceeds to the next step to determine if any improvement can be obtained.



i	:	1	2	3
k_i^+	:	3	1 , 3	1
$k_{j=4}^-$	:	-	1	-
k_i^+	:	3	1 , 3	1
$k_{j=5}$	:	3	1 , 3	1
k_i^+	:	3	1	1
$k_{j=6}^+$	:	3	1	1
k_i^+	:	3	1	1
t_i^+	:	-	-	-
Π_i	:	-10	+1	-9

Fig. 8. Computation of Π_i in Step 2 of the algorithm.

Choose $i^* \in \mathrm{NI}$ based on Eq. (19)

$$\Pi = \operatorname*{Max}_{i \in \mathrm{NI}} \Pi_i = \Pi_{i^*}. \tag{19}$$

If $\Pi \leqslant 0$ go to Step 3,

Set QTERM = 1, K = $k^+_{i^*}, T = t^+_{i^*}$,

$$\sigma = \mathrm{Min}\begin{cases} \delta_k \text{ if } k \in K \text{ and } c_{ij} \leqslant a_j, & \gamma(k,j) \in A^m_M \\ s(n_j j) \text{ if } k \in K \text{ and } c_{ij} > a_j, & \gamma(k,j) \in A^m_M \\ q(\zeta_t, t) \text{ if } t \in T. & \end{cases} \tag{20}$$

Create δF^m as follows:

$$\delta Q(O, i^*) = +\sigma, \tag{21}$$

$$\delta q(O, k) = -\sigma, \quad k \in K, \tag{22}$$

$$\delta q(k, j) = -\sigma, \quad k \in K \text{ and } (k, j) \in A^m_M, \tag{23}$$

$$\delta q(\zeta_j, j) = -\sigma, \quad j \in T, \tag{24}$$

$$\delta q(i^*, j) = +\sigma, \quad j \in T \tag{25}$$

$$\delta q(i^*, j) = +\sigma, \quad (K, j) \in M^m \text{ and } c_{i^*j} \leqslant \alpha_j, \tag{26}$$

$$\delta q(\zeta_j, j) = +\sigma, \quad (K, j) \in M^m \text{ and } c_{i^*j} > \alpha_j. \tag{27}$$

The δF^m created in either Steps 1 or 2 is an incremental change in the solution which improves the objective function. The subgraph δF^m has positive as well as negative capacities corresponding to increases and decreases in the current solution.

In the final step, which is Step 3, a check is introduced to discover whether the present iteration has produced an improved solution, and if not, the algorithm terminates. If an improved solution has been achieved, the new capacity subgraph F^{m+1} is created by composing $F^m + \delta F^m$. The corresponding marginal capacity subgraph M^{m+1} and surplus capacity subgraph S^{m+1} are set up and the algorithm proceeds to another iteration.

Step 3: If QTERM = 0 Go to END

Create F^{m+1} according to

$$Q(O, i) = Q(O, i) + \sigma(O, i) \quad \forall i \in \mathrm{NI}, \tag{28}$$

$$q(i, j) = q(i, j) + \delta q(i, j) \quad \forall i \in \mathrm{NI}, \ j \in \mathrm{NJ}, \tag{29}$$

$$q(O, j) = q(O, j) + \delta q(O, j) \quad \forall j \in \mathrm{NJ}. \tag{30}$$

Create M^{m+1} and S^{m+1} as creating M^0 and S^0 in Step 0.

Set $m = m + 1$

Go to Step 1

END

To show that the algorithm is heuristic one and does not always terminate with an optimal solution a counter-example is given in Fig. 9. The missing response arcs have a very high cost (one supposes infinite) so that they will never appear in an optimal solution. Clearly, the initial feasible solution to this problem will allocate 100 units of capability at each of locations 1, 4 and 5. The acquisition cost for this solution is 15,000, while the response cost is 6000. However, a better solution exists with 100 units of capability allocated at each of locations 2 and 3. The acquisition cost is now only 10,000 while the response cost increases to 6600.

The heuristic procedure cannot find this improved solution because it cannot examine simultaneous increases in capability at two or more locations. In examining the possibility of in-

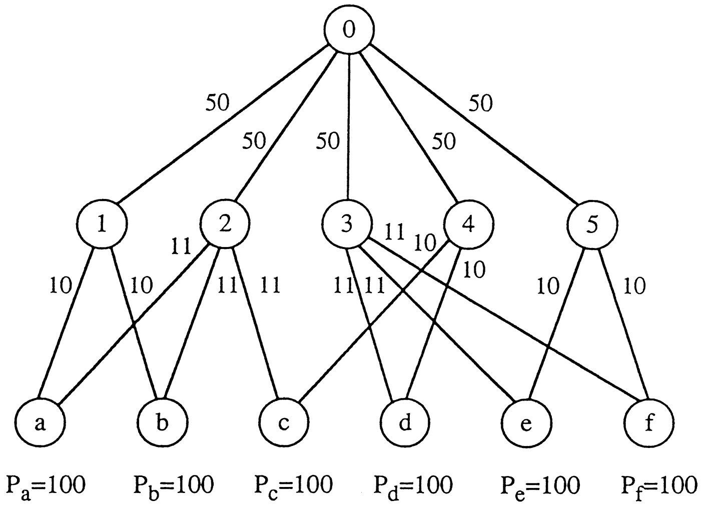


Fig. 9. Counter example where optimality is not reached.

creasing the capability at location 2 alone, it finds no improvement. Reducing the capability at location 1 alone is not sufficient since there are no acquisition cost savings. On the other hand, reducing the capability of locations 1 and 4 involves the infinite cost arc (2, d). A similar argument applies to increases at location 3 alone. The only possibility of increasing the capability at locations 2 and 3 is for them to be increased simultaneously, thereby, avoiding the infinite cost response arcs (2, d) and (3, c). If the algorithm is extended to include this simultaneous operation, the amount of computations will increase very rapidly (say, in an exponential manner). As will be shown later, such an extension cannot be justified. Nonetheless, the information given in the example in Fig. 9 is unlikely to exist in reality. Response arc costs are likely to be more or less close to one another. Hence it is expected that the algorithm will usually terminate at or close to the optimal solution.

4. The algorithm incorporating fixed costs

The heuristic solution to the network synthesis problem does not consider the fixed costs FX_i of establishing the facility at location i. If these costs are zero then the algorithm presented thus far is sufficient. However, often these costs are not zero, and therefore, there is a need to devise a solution scheme which permits the use of the algorithm with these costs. In this section, an implicit enumeration technique is developed to allow for such a solution to be reached efficiently.

Define a vector $X = (x_1, \ldots, x_i, \ldots, x_h)$where,

$$x_i = \begin{cases} 1 & \text{if location } i \text{ is a candidate location,} \\ 0 & \text{otherwise.} \end{cases} \tag{31}$$

The vector $X'_i = (x'_1, \ldots, x'_i, \ldots, x'_h)$ is a successor to $(x_1, \ldots, x_i, \ldots, x_h)$

$$\text{if} \quad x'_i \leqslant x_i, \quad i = 1, \ldots, h \tag{32}$$

and

$$x'_i < x_i \quad \text{for some } i, \ i = 1, \ldots, h. \tag{33}$$

The vector X' is an immediate successor to X if

$$\sum_{i=1}^{h} x_i = \sum_{i=1}^{h} x'_i + 1. \tag{34}$$

Define $V(x_1, \ldots, x_i, \ldots, x_h)$ as the objective function value corresponding to the solution obtained by the heuristic algorithm with the location set

$$P = \{i : x_i = 1\} \tag{35}$$

and define

$$F(x_1, \ldots, x_i, \ldots, x_h) = V(x_1, \ldots, x_i, \ldots, x_h) + \begin{cases} \mathrm{FX}_i, & x_i = 1, \\ 0, & x_i = 0. \end{cases} \tag{36}$$

Theorem. *$V(x_1, \ldots, x_i, \ldots, x_h)$ is a lower bound on $F(x'_1, \ldots, x'_i, \ldots, x'_h)$ for all successors X' to X.*

Proof. Since our problem is a minimization

$$\begin{aligned} V(x_1, \ldots, x_i, \ldots, x_h) &\leqslant V(x'_1, \ldots, x'_i, \ldots, x'_h) \\ &\leqslant F(x'_1, \ldots, x'_i, \ldots, x'_h). \end{aligned}$$

The algorithm first solves the problem including all locations $i \in \mathrm{NI}$ in P. This produces an upper bound $F(1, \ldots, 1)$ on the solution's objective function value. The algorithm is then applied successively to each of the h immediate successors to $X = (1, \ldots, 1)$. If the objective function value $V(x'_1, \ldots, x'_i, \ldots, x'_h)$ of any successor is greater than the current upper bound $F(X)$, then this successor and all its successors are eliminated from further consideration. This procedure is termed fathoming.

For all successors that are not fathomed the $F(x'_1, \ldots, x'_i, \ldots, x'_h)$ are computed, and the upper bound is updated if any $F(x'_1, \ldots, x'_i, \ldots, x'_h)$ is less than the current upper bound. The procedure is repeated until no further successors are left, at which point the current upper bound is the optimal solution.

The success of the algorithm naturally depends on the tightness of the bounds generated and the latter depends on the level of the fixed costs FX_i in relation to the solutions $V(x)$. If the fixed costs are large in comparison to the $V(x)$, the bounds may not be good. In the example of the oil spill case, the fixed costs are relatively low and hence our method produces reasonably tight bounds.

The formal algorithm for fixed costs is presented below followed by an example in Fig. 10.

Intialization: The algorithm begins with $P1 = 1, 2, \ldots, \mathrm{NI}$ and $P = \mathrm{NI} - P1 + 1$; then defining a set of locations indexed I where $I = 1, 2, \ldots, \binom{\mathrm{NI}}{P}$. Determine the corresponding vector X, and if X is not eliminated go to Step (1). Apply Fathom (X).
Step (1): Evaluate $V(X)$. If P is less than NI go to Step (2). Calculate $F(X) = V(X) + x_i \cdot \mathrm{FX}_i$; $F^* = F(X)$.
Step (2): If $V(X)$ is less than F^* go to Step (3). Apply Fathom (X) and END.
Step (3): $F(X) = V(X) + x_i \cdot \mathrm{FX}_i$. If $F(X)$ is less than $F^*, F^* = F(X)$.

The routine Fathom (X) eliminates all successors of X. This process for $\mathrm{NI} = 5$ appears in Fig. 10.

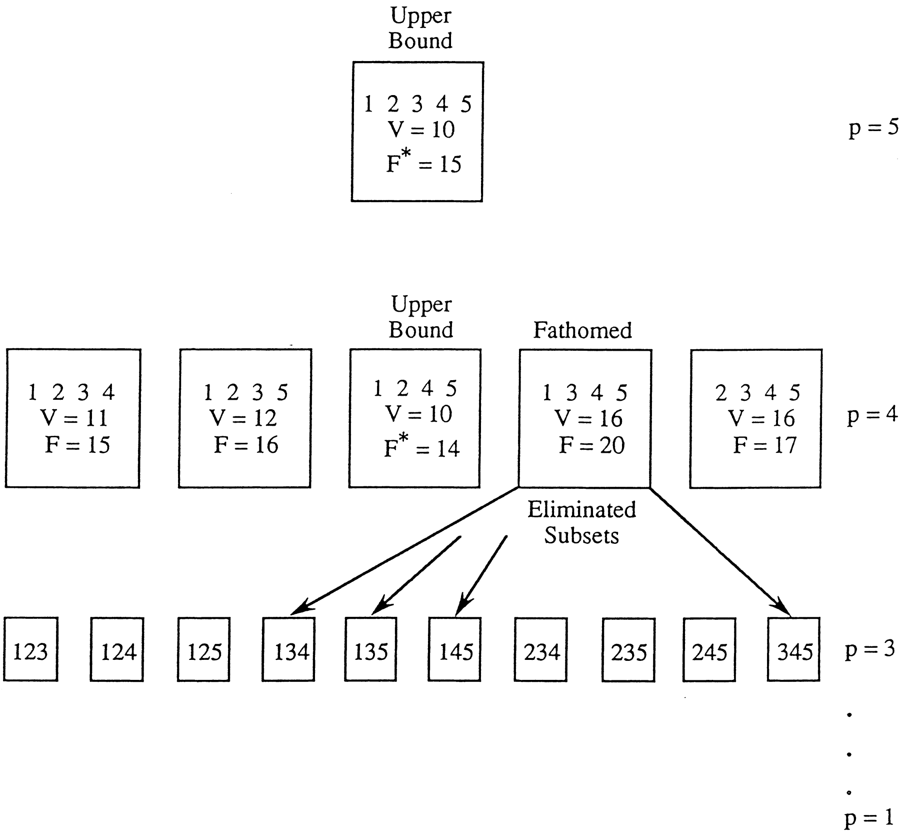


Fig. 10. Implicit enumeration algorithm for fixed costs.

5. Incorporation of response time stipulation

In the oil spill case, as can be expected, there are response time stipulations. These response time stipulations can be incorporated within the heuristic procedure. We can define $X_j(r)$ as the set of candidate locations "covering" the risk point j for a given response time stipulation r. The response time stipulation requires that a response of level v_j be available within r hours. We can also define an additional risk point j' corresponding to j for which $p_{j'} = v_j$, and modify the arc costs according to

$$c_{ij'} = \begin{cases} c_{ij}, & i \in X_j(r), \\ \infty, & \text{otherwise} \end{cases} \tag{37}$$

and

$$c_{Oj'} = \infty. \tag{38}$$

Based on these definitions, the response time stipulation is automatically accounted for in the solution. The infinite arc costs for $i \notin X_j(r)$ ensure that response to j will be restricted to arcs satisfying the response time constraints (if no such arc is available the problem is not feasible). By setting $c_{Oj'}$ to infinity we ensure a total response of v_j.

Table 1
Computational comparison heuristic vs LP

	NI	NJ	Number of constraints	CPU time (s)		Objective function		
				Heur.	LP	Heur.	LP	Deviation (%)
1	3	48	192	4.59	14.65	5.7448	5.6922	0.92
2	3	51	204	4.78	16.32	6.8299	6.7674	0.92
3	3	54	216	5.32	17.36	6.8392	6.7717	0.99
4	3	57	228	5.50	20.06	6.8593	6.7917	0.99
5	4	48	140	5.19	20.11	5.7383	5.6832	0.97
6	4	51	255	5.65	22.85	6.8233	6.7584	0.96
7	4	54	270	6.05	24.34	6.8326	6.7627	1.03
8	4	57	285	6.77	26.57	6.8526	6.7827	1.03
9	5	48	288	5.55	25.75	5.7383	5.6832	0.97
10	5	51	306	6.40	29.36	6.8224	6.7572	0.96
11	5	54	324	6.90	32.00	6.8316	6.7616	1.04
12	5	57	342	7.30	36.66	6.8517	6.7816	1.03
13	32	54	1782	15.11	–	7.3465	–	–
14	32	57	1824	16.52	–	7.3668	–	–
15	40	54	2214	12.10	–	6.8005	–	–
16	40	57	2337	12.77	–	6.8202	–	–

6. Computational experience and concluding remarks

The heuristic algorithm developed has been subjected to some computational testing, and the results are presented here. These results refer to various abbreviated forms of the problem generated by real-life data. The abbreviated forms consist of parts of a single 40 × 57 network of the US New England zone [7].

The network synthesis, as it is mentioned in the introduction, can be solved optimally by the LP technique as it is formulated, for example, in [7]. On the other hand, the LP procedure cannot handle large-scale problems like the oil spill problem in the US New England zone. The LP approach leads to extremely large problems because of the $|\mathrm{NI}| \times |\mathrm{NJ}|$ response constraints. A problem involving 20 possible equipment locations, 5 equipment types and 20 risk points, with three discrete spill volumes, would entail solving a LP problem having more than 6000 constraints. In addition, if there were fixed costs for establishing facilities at any of the locations, the solution procedure would involve solving such a problem many times over.

The results of several feasible networks are compared in Table 1, which gives the problem size, the LP solution, the heuristic solution and their respective CPU times in seconds. The LP codes used are mentioned in [4], and are codes considered standard for research applications. The results indicate considerable savings in CPU time in the case of the heuristic solution. As can be seen in Table 1, after the 12th network, the LP procedure could not handle the large-size problems. The highest deviation observed was 1.04%. Importantly, the ratio of heuristic CPU time to LP CPU time appears to decrease with increases in problem size. Also, the deviation appears to be reasonably stable. Clearly there is a room for considerable work on the computational aspects of the heuristic procedure which may open up opportunities for future interesting results. It is also worthwhile mentioning that the problem formulated in this research may be found adequate for other real-life problems, and may enhance the use of the developed algorithm.

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