A model-based approach for tactical decision making in oil spill response

Konstantinos G. Gkonis[1], Nikolaos M.P. Kakalis*[2], Nikolaos P. Ventikos[1], Yiannis Ventikos[2], Harilaos N. Psaraftis[1]

[1]Laboratory for Maritime Transport, School of Naval Architecture and Marine Engineering, National Technical University of Athens
9 Heroon Polytechniou Str. 157 79 Zografou, Greece

[2]Fluidics and Biocomplexity Group, Department of Engineering Science
University of Oxford, Parks Road, Oxford OX1 3PJ, United Kingdom

Abstract

In this paper we propose an integrated approach for tactical decision making in oil spill response based on information on the oil fate. The optimization problem is sequentially coupled with a dynamic mathematical model that provides estimates of the oil spill fate at the contact time of the spill with the response means. The model consists of a set of differential and algebraic equations that describe the spill dynamics as these are affected by spreading and weathering. To solve the tactical problem, an integer optimization problem is formulated where the objective is to minimize the total costs considering the response system costs and the resulting reduction in the spill damage. Appropriate constraints on equipment operability and capacity, response time, supply and mother vessel use are set. The use of the methodology is illustrated via its application in a realistic case where the response means considered is the EU-MOP system (Elimination Units for Marine Oil Pollution).

Keywords

Oil spill response; tactical decision-making; oil fate modeling

1. Introduction

The oil spill decision-making process is distinguished in three hierarchical levels: the strategic, the tactical and the operational (Anthony, 1965 and Psaraftis & Ziogas, 1985). The tactical level, where the focus of this paper is, determines the actions required to respond to a specific spill, whereas the strategic level is concerned with potential future spills, and the operational determines the detailed actions to be taken on the scene of a specific spill. More specifically, the tactical level problem can be described as follows. A spill of known characteristics occurs. In the broader area of the incident a number of response facilities exist, that are usually located at ports. These facilities are equipped with specific quantities and types of oil response equipment of certain characteristics (oil recovery capacity, etc). The decision-maker needs to determine which facilities to dispatch units from as well as the types and quantities of the units to be sent. This is usually done either heuristically or via the solution of an optimization problem.

In the model presented in this work, the approach adopted is to respond optimally on a cost basis. In this case, the decision-maker's objective is to minimise the total cost, by balancing the response (system) costs and the spill damage costs with the introduction of a relative weight coefficient. Other approaches could set as primary objective to minimise the response time or maximise the coverage of spill (e.g. Belardo et al, 1984).

In these problems, an as accurate as possible description of the spill along with other data provided to the optimization, play a significant role in determining the optimal response decision.

In this work an integrated approach for tactical decision making in oil spill response is proposed, in which the optimization problem is sequentially coupled with a dynamic mathematical model that provides estimates of the oil spill fate at the contact time of the spill with the response means. Given the characteristics of the specific oil spill that define the initial conditions and oil-specific model parameters, the dynamic model is simulated up to the spill/response contact time. The current volume and area of the spill are exported and become an input to the

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* Corresponding author: nikolaos.kakalis@eng.ox.ac.uk; Tel: +44 1865 273921; Fax: +44 1865 273010

optimization problem. This is then solved using the branch-and-bound method. In Figure 1 we provide a schematic overview of the proposed approach.

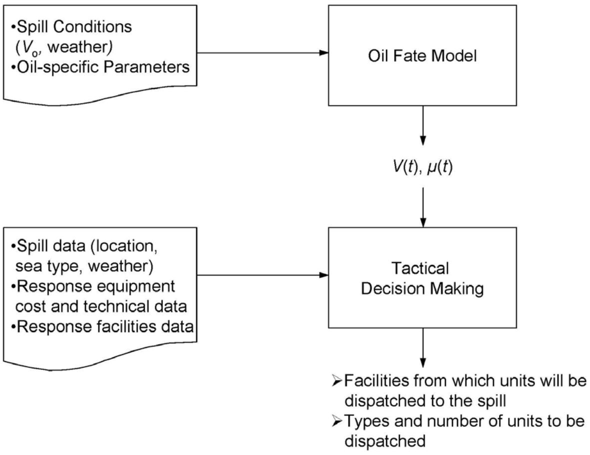

Fig. 1: Schematic overview of the proposed method.

The use of the methodology is illustrated via its application in a realistic case where the response means considered are the EU-MOP (Elimination Units for Marine Oil Pollution). This is a robotic swarm approach developed in the context of the EU-MOP project which is funded by the European Commission under FP6-2003-516221. The EU-MOPs are autonomous unmanned oil-cleaning robot vessels of Monocat and Catamaran types and of sizes ranging in length from approximately 1 to 3 m (Fig. 2), each one equipped with a number of sensors (for navigation, oil detection etc.). EU-MOP units will operate as a swarm to clean a spill (Fig. 3).

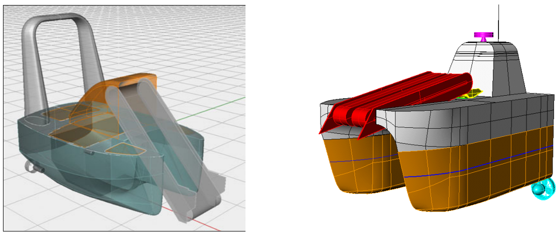

Fig. 2: Monocat and Catamaran type EU-MOP units.

Fig. 3: EU-MOP swarm in operation.

The control station of a swarm will be located either on the shore (e.g. close to a refinery) or on a mother ship (Fig. 3), which will transport the response units to the spill site, with the assistance of other supply (transport) vessels, if required by the total number of units to be dispatched. The EU-MOP tactical decision maker will have the task of determining from which facilities to dispatch response equipment to a spill site and, moreover, the types and quantities of the units to be dispatched. The EU-MOP tactical response command and information flows are shown in Figure 4.

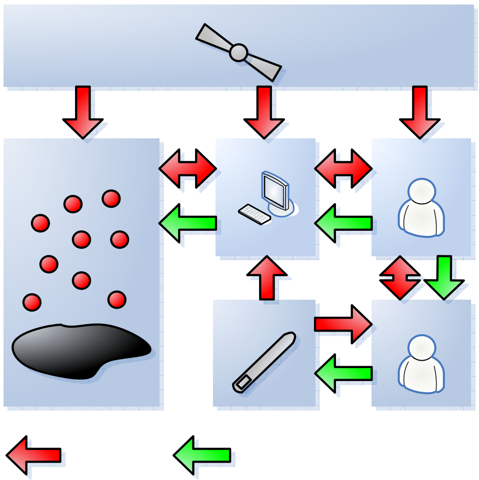

Fig. 4: EU-MOP command and information flows.

The rest of the paper is organised as follows. After a brief review of the relevant literature in Section 2, the mathematical formulation of the oil fate model is presented in Section 3 and that of the tactical decision making problem in Section 4. Specificities of the models are given in Section 5. In Section 6, an illustrative example of the model is presented and then Section 7 concludes the paper.

2. Background

The present paper refers to the tactical problem of the oil spill decision-making process that is part of an integrated work in the EU-MOP project which also covers the strategic problem. In this section, reference is made to the literature that provided the necessary background to this work and an understanding of the accumulated knowledge and experience in the field. Many of these papers address the strategic problem, the solution of which is an input to the tactical problem. As the strategic problem was developed in parallel to and as an aggregation of tactical problems, such papers are also discussed next. However, the comments are restricted to points mainly relevant with the modelling of the tactical level problem.

A seminal paper about the strategic planning of oil spill response is the one by Psaraftis et al. (1986). The paper deals with the strategic aspect of the oil spill response problem. It presents the development of a model for allocating appropriate levels and types of clean-up capability to respond to future oil spills among points of high oil spill potential. The present work has adopted many elements from this model. For example, the objective of the problem is to minimise the expected algebraic sum of the response system costs and the costs due to damages from spills, the latter balanced with a user-specified “weight” coefficient. Also, many problem parameters have been adopted as suggested in this paper (various cost & technical parameters, etc.). However, the presented model addresses a more complicated response infrastructure that requires the dispatch of a mother vessel and possibly more than one supply vessels for the transportation of response units from the facilities to a spill site (in accordance with the EU-MOP concept).

The paper by Psaraftis & Ziogas (1985) provided an example of a tactical level decision-making algorithm. This paper describes a deterministic methodology for the optimal allocation of resources for cleaning up a specific spill after its occurrence is made known. This is also the rationale of the tactical algorithm in the presented model. Moreover in the present work, the approach in Psaraftis & Ziogas' paper regarding the hierarchical levels of the oil-spill decision-making process (i.e. strategic, tactical and operational), the distinction of the respective decision variables and their interaction have been followed (see also Anthony, 1965). For example, equipment acquisition costs and other costs that have been committed at the strategic decision level are sunk costs and are not considered in the tactical problem.

On the other hand, the present model differs structurally from Psaraftis & Ziogas' one. The major difference is that their model is part of a broader model ("the MIT oil spill model"), where spill incidence, damage assessment, the strategic model and other components are equally distinct parts and therefore external to the tactical problem. In the end, Psaraftis & Ziogas construct a tactical model solved at discreet time steps using a dynamic programming solution algorithm, with numerous inputs furnished externally. Instead, the presented tactical problem is a stand-alone model (e.g. with a built-in damage assessment algorithm) with an original and simplified structure using a mixed-integer programming solution algorithm. This model also forms part of a broader simulation tool that addresses the strategic decision-making of the oil spill response procedure. In that simulation, the tactical model is recalled and run numerous times in the search for a solution to the strategic problem.

Belardo et al. (1984) presented an alternative approach to oil-spill response decision-making. The objective in this model is not to minimise a function of cost, but to maximise the overall probability of covering an oil spill incident. The notion of "coverage" is defined in terms of the availability of the needed resources within a critical time; that is before the spill hits the shore. The model can accommodate a budget constraint but does not consider the trade-offs between the spill response and damage costs on a cost/benefit basis. An alternative model according to this rationale is also under development and will hopefully be used at a later stage for verification and comparison purposes. From Belardo et al.'s model, the damage potential assessment rationale has been adopted. Accordingly, a spill incident is placed into one of three distinct groups of (ranked) damage potential, following the assessment of its impact on various target categories.

A paper by Iakovou et al. (1996) has been considered and certain similarities with the presented model can be identified. First, the solution of the strategic problem is not addressed independently of the tactical problem. Strategic level decisions are evaluated by taking into account their impact on post oil-spill decisions. Also, an integer programming algorithm is used, with the integrality relaxed, as in the presented model. Another similarity is the critical time to respond to a spill incident that is introduced as a constraint. However, the above model requires significant data preparation work. Transportation costs are assumed to comprise both clean-up and damage costs expressed in the model through a ratio of unit cost to unit time parameter. Our model is more detailed and explicit in calculating these costs and introducing them in the objective function of the optimisation.

Other relevant works include Charnes et al. (1979), Charnes et al. (1976), and Srinivasa and Wilhelm (1997). Ceder et al. (2001) is an extension of the work in Psaraftis et al. (1986), which was discussed-above, and it introduces a heuristic algorithm for dealing with large-scale problems. Also, a paper by Iakovou et al. (1994) reviews the models that had been developed up to that time regarding oil-spill response planning. Their commentary covers historical data analysis, strategic decision-making, tactical decision-making, and operational decision-making. Alidi (1993) undertakes a similar review.

As the focus of this work is to enhance tactical decision making via its coupling with predictions of the oil spill fate, we are turning our attention to this direction.

Realistic information on the size and condition of the spill in time plays an important role in the accuracy of the tactical response optimisation with a profound effect on the operations economics. Therefore, there is a need for a mathematical representation that will provide predictions of the spill fate. However, mathematical modelling of oil spill systems is non-trivial. A variety of complex physical, chemical and biological phenomena act simultaneously. The transformation processes depend on the initial oil properties, the spilled amount, hydrodynamics, climatic and sea conditions. All these factors vary with time, and the way they determine the fate of the spilled oil and the subsequent consequences is complex. Due to the criticality of the issue a lot of modelling work can be found in the literature, mainly based on empirical and semi-empirical developments (Brebbia 2001). It is estimated that over 50 oil spill models have been developed but there are only a few that are used extensively in practice today (Nasr & Smith 2006), like the ADIOS (National Oceanic and Atmospheric Administration - NOAA), S.L. ROSS (S.L. ROSS Environmental Research Ltd, Canada), OSIS (British Maritime Technology Ltd, UK), SINTEF OWM (SINTEF Group, Norway) and OILMAP (ASA Consulting Ltd, USA).

Regarding the integration of oil spill response with spill fate modelling approach of this paper, a relevant work is embedded in the OSCAR system by Reed et al. (1995). The OSCAR system was designed to meet, among other needs, the establishment of quantitative measures of effectiveness for regulatory and management decision-making, cost-benefit analysis, rationalization, and optimization for equipment purchase and disposition and the evaluation of alternative oil spill response strategies and logistics. Components of OSCAR include a three-dimensional oil and chemical spill model and a simulation model for strategic analysis of oil spill response actions. On the other hand, OSCAR is a complex system of modelling tools, rather than a stand-alone simulation tool for immediate decision-making support.

Galt (1998) showed the importance of uncertainty in oil spill modelling and pollution confrontation. In this context, he focused on the preliminary coupling of response strategies with oil spill trajectory and fate parameters, in the scope of the formulation of efficient operations.

Elliot and Jones (2000) addressed the need for operational forecasting during oil spill response. They discussed the possibility regarding the integration of certain environmental aspects (e.g. wind data) in the framework of operations for pollution counteraction, and in terms of equipment suitability in relation to local prevailing conditions.

Etkin (2001) referred to a method for oil-spill response cost estimations based on labour/equipment requirements. This approach involves reviews of historical case studies to estimate worker-days and the equipment required for oil spill recovery operations. These estimations are coupled with information on the general behaviour of different oil types and amounts using the National Oceanic and Atmospheric Administration's (NOAA) Automated Data Inquiry for Oil Spills (ADIOS) software. The oil behaviour data are used to modify the response work estimates by taking into account slick spread, dispersion, evaporation, and emulsification by the time mechanical operations start.

Eide et al. (2007) presented a dynamic environmental risk model that can help in preventing oil spill from drift grounding accidents of oil tankers. The model's main purpose is to assess the environmental risk of a drift grounding accident concerning a given ship, location, and weather conditions. The paper raises interesting questions regarding the uncertainties surrounding risk estimates, the use of historical data and the impact modelling. In the end, this assessment can be useful for a risk-based positioning of tugs along the coast, i.e. the placement of tugs where they can be most effective. Hence, it may function as a decision support tool facilitating strategic tug positioning, and in this respect it is also relevant to the work presented in this paper. An important note by the authors is that the intended use of the model is decision support, not decision making, as ultimately, all decisions must be made by qualified operators.

In the following sections we provide analytical details on the methodology presented in this paper and the embedded models.

3. Oil fate model

In this section we present the oil fate model considered in this work. Although any fate model and in any form can, in principle, be used in the methodology presented here, we choose to employ an equation-oriented approach following the implementation of Kakalis and Ventikos (2007).

The dominant processes that cause significant short-term changes in oil characteristics over time are spreading, evaporation, dispersion and emulsification. They all occur progressively as oil weathers at rates which depend on the oil composition and the prevailing temperature and wind speed. Spreading reduces oil thickness and evaporation increases the flash point, pour point density and viscosity. Emulsification might increase significantly the fluid volume, and the density of spilled oil and its viscosity usually by two or three orders of magnitude. In this section we present a mathematical representation of the dynamic behaviour of weathering oil spill. The main modelling assumptions are that the oil slick is considered as a bulk system and does not account for the individual chemical components that are involved in the various processes.

The rate that oil evaporates from the sea surface is given by Stiver and Mackay (1984) as:

$$\frac{dF}{dt} = \frac{K_{ev}A}{V_0}\exp\left(A_{ev} - \frac{B_{ev}}{T}\left(T_z + T_g F\right)\right) \tag{1}$$

where F is the volume fraction of evaporated oil, V_0 is the spilled volume (m^3), A is the area of the slick (m^2), T is the oil temperature (K) and t refers to time (s). The evaporated oil volume, V_{ev}, is then given as $V_{ev}=V_0F$. K_{ev} is a mass transfer coefficient given by Buchanan and Hurford (1988):

$$K_{ev} = 2.5\times 10^{-3} U^{0.78} \tag{2}$$

where U is the wind speed (m/s). A_{ev} and B_{ev} in equation 1 are empirical constants set at 6.3 and 10.3 respectively (NOAA 1994), and T_z, T_g are the initial boiling point and the gradient of the modified oil distillation curve respectively. These are given as functions of the oil API[1] degree (NOAA 1994); for crude oils:

$$T_z = 532.98 - 3.1295 API \tag{3}$$

$$T_g = 985.62 - 13.597 API \tag{4}$$

and for refined products:

$$T_z = 645.45 - 4.6588 API \tag{5}$$

$$T_g = 388.19 - 3.8725 API \tag{6}$$

The rate of dispersion into the water column of floating substances at the sea surface is given by:

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[1] API stands for American Petroleum Institute

$$\frac{dV_d}{dt} = 2\times 10^{-8} U^2 \left(V_0 - V_{ev}\right) \qquad (7)$$

where V_d is the volume of the dispersed oil (m3). This expression is based on data from the Ekofisk accident.

In emulsification, water droplets are entrained in the oil. This causes significant changes in the volume, density and especially viscosity of the slick. Crude oil will emulsify when the wax and asphaltene content reach 5% of the mass of the oil. Light refined products, in general, are not expected to emulsify since they do not contain the right hydrocarbon components to stabilize the water droplets. The corresponding variations on oil properties are also given in the latter paper. The dynamics of the emulsification process are given by (Mackay et al. 1980):

$$\frac{dY}{dt} = K_{em}(1+U)^2\left(1-\frac{Y}{Y^f}\right) \qquad (8)$$

where Y is the volume fraction of water in the emulsion with Y^f being its final value and K_{em} is empirical constant between 1×10-6 and 2×10-6.

The rate of spreading is given as (Mackay et al. 1980):

$$\frac{dA}{dt} = K_s A^{-1}\left(V_0 - V_{ev} - V_d\right)^{4/3} \qquad (9)$$

where K_s is a parameter of value 150 s^{-1}. This type of models assumes that the oil spillage is instantaneous. In this case, the initial spreading of the spill can be attributed to combined gravity/viscosity phenomena and the initial area of spilled oil, A_0 (m^2), is determined by Fay (1969):

$$A_0 = \pi \frac{k_2^4}{k_1^2}\left(\frac{\Delta g V_0^5}{\nu_w}\right)^{1/6} \qquad (10)$$

where g is the gravity acceleration, $\Delta=(\rho_w-\rho_0)/\rho_w$, with ρ_w being the seawater density and ρ_0 the fresh oil density, ν_w is the water kinematic viscosity and k_1, k_2 constants of value 0.57 and 0.725 respectively (Flores et al. 1998).

The general mass balance that gives the remaining volume of oil, V (m^3), due to weathering can be written as:

$$V = \left(V_0 - V_{ev} - V_d\right)\frac{1}{1-Y} \qquad (11)$$

and the thickness of the oil spill h (m) is derived by $V=hA$.

Additional algebraic equations can be used to describe the time variation of physical properties of interest. The dynamic viscosity of the oil, μ (cP), is given by (Mackay et al. 1980):

$$\mu = \mu_0 \exp\left[C_T\left(\frac{1}{T}-\frac{1}{T_0}\right)\right]\exp\left(C_1 F\right) \qquad (12)$$

where μ_0 is the starting oil viscosity (cP) at reference temperature T_0 (K), and C_T, C_1 are empirical constants. C_T is usually set at 5000 K and C_1 varies between 1 and 10 with oil type, with higher values corresponding to more viscous products. Note that the first term of equation 12 corresponds to the influence of temperature and the second to that of evaporation. The final emulsion viscosity is determined by (Mooney 1951):

$$\mu_{em} = \mu_0 \exp\left[\frac{2.5Y}{\left(1-Y^f Y\right)}\right] \qquad (13)$$

Equation 13 imposes a significantly more pronounced increase on viscosity than equation 12. Similarly, the density, ρ (kg/m^3), of the emulsified oil is given as (Buchanan and Hurford 1988):

$$\rho = Y\rho_w + (1-Y)\left(\rho_0 + Y^f F\right) \qquad (14)$$

The model described above comprises a DAE system[1], the numerical solution of which derives the weathering prediction of specific oil under given environmental conditions.

4. Tactical response decision making

Let I be the set of available response (stockpiling) facilities in the vicinity of a spill site, each with up to E types of response equipment. At each response facility i, $i \in I$, N_{ie} units of type e, $e \in E$, are stored (cf. Figure 5).

An oil spill incident of known characteristics occurs. The tactical level decision-making determines the amount of units x_{ie} of each type e that should be dispatched from each facility i to the spill site. In addition, it determines whether a mother vessel will be dispatched to the spill site and whether additional supply vessels will be dispatched and from which response facilities in accordance to the EU-MOP concept. Hence, two additional decision variables are introduced: $SV_i \in \{0,1\}$ a binary variable to determine whether a supply vessel is dispatched from facility i to the spill site, and $MV \in \{0,1\}$ a binary variable to determine whether a mother vessel is dispatched to the spill site.

The above problem is formulated as an optimisation program by applying the mixed-integer programming theory. The values of the decision variables that meet certain constraints and minimise a total cost objective function by balancing the system and the potential damage costs are the solution to the problem.

The tactical problem defined here is an integer program (IP), as the decision variables take integer values, a subset of which are binary (those referring to the mother/supply vessels). In practice, integrality is relaxed for the non-binary decision variables and the corre-

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[1] Differential and algebraic equations system.

sponding mixed-integer programming (MIP) problem is solved using the branch-and-bound method (Hillier & Lieberman, 1995). For each possible set of decision variables, a new value for the objective function is calculated. When the optimisation is finished the optimal values of the decision variables are found, which yield the best (cost) value for the objective function under the given constraints.

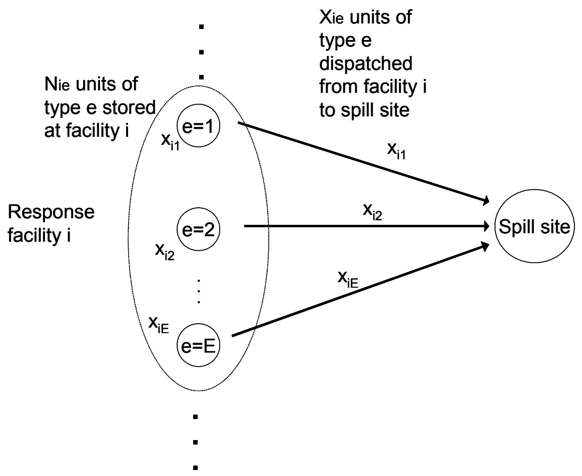

Fig. 5: Problem definition.

4.1 Model parameters

In the following, the explanation of the parameters used in the mathematical model is given.

General parameters:

$o \in O$: Oil type of the spill incident;

$w \in W$: Weather conditions category of the spill incident;

$s \in \{0,1\}$: Direction of the spill (away from the shore or towards the shore);

u_e (tons / h): Oil recovery capacity of 1 unit of type e equipment;

RE_e : Recovery efficiency rate of type e equipment;

T_{ie} (h): Available time for on-site cleaning operations at the spill site for type e units dispatched from i;

TT_i (h): Total time of operations (travel to + on-site cleaning + return) at the spill site for units dispatched from i;

MTT (h) = median (TT_i) : Assumption for the total time of operation of the mother vessel dispatched to the spill site;

$TMIN$ (h): Minimum available time that justifies dispatching equipment for on-site cleaning operations;

v (tonnes) : Size of recorded spill;

CC: Desired coverage coefficient of the spill incident (CC=1 corresponds to the exact size of the spill);

WC: Weathering coefficient that determines the amount of oil that is left available (for collection by the response equipment) because of weathering effects;

WC_i: Weathering coefficient determining the remaining spill size (because of weathering effects only) when the response equipment from facility i arrives at the spill site;

OV_i (cSt): Oil viscosity when response equipment from facility i arrives at the theatre of the spill;

VL_e (cSt): Operational limit of equipment type e regarding the viscosity of oil to be collected;

N_{ie} : Maximum number (storage capacity) of type e units at facility i;

DL_{ei} : Variable to determine whether type e from facility i is operational with respect to the oil viscosisty, when e arrives at the spill site from facility i;

DC_e : Variable to determine whether type e is operational with respect to the oil type, the weather conditions and the sea type at the spill site;

DT_{ie} : Variable to determine whether type e units can be dispatched from facility i to the spill site within the time limitations;

DL_{ie} : Variable to determine whether type e from facility i is operational with respect to the oil viscosity, when e arrives at the spill site from facility i.

Cost parameters:

SDC: The system / damage costs coefficient;

CT_{ie} (euros): Cost of transporting 1 unit of type e equipment from i to the spill site;

b_e (euros/h): Clean-up (operational) cost of 1 unit of type e equipment;

CO_o (euros/tonne): Cost of oil of type o;

DP : Damage potential coefficient assigned to the damage group where the spill incident is placed (explained later);

CSV (euros / h): operational cost of supply vessel (SV);

$ECMV$ (euros / h): extra operational cost of mother vessel (MV) (compared to SV).

4.2 Objective function

The aim is to respond to the specific spill in an optimal way. In this model, the approach adopted is to respond optimally based on a cost criterion. In this case, the decision-maker's objective is to minimise the total cost by balancing the response (system) costs and the spill damage costs with the introduction of a relative weight coefficient.

In general, more expenses devoted in response (an increase in response costs) will result in faster collection of more oil, i.e. in a reduction of the spill damage costs. The overall spill damage costs are the non-response costs (that would be incurred if no response took place) minus the damage costs that would result from the amount of oil that is collected. A weight coefficient is used to determine the relative value of response vis-à-vis damage costs, as it will be later explained.

The objective function to be minimised is the total cost consisting of the response costs, the response benefit and the non-response costs[1]:

$$\text{Total cost} = (\text{Response costs}) + (\text{Response Benefit}) + (\text{Non-Response costs}) \quad (15)$$

The response costs are the transportation and clean-up costs for dispatching a number of units to the spill site, plus the operational costs of the mother/supply vessels. The response benefit (negative costs) results from the (partial or total) recovery of the spilled oil, which corresponds to a reduction in the total damage (potential) costs of the spill. The non-response costs are the expected total damage (potential) costs that would result from the spill incident, if no response took place. More specifically, the objective function can be mathematically expressed as:

$$\begin{aligned} &\text{Total Cost} = \\ &\sum_i \sum_e x_{ie} \cdot (CT_{ie} + b_e \cdot T_{ie} - SDC \cdot u_e \cdot T_{ie} \cdot RE_e \cdot CO_o \cdot DP) + \\ &\sum_i SV_i \cdot TT_i \cdot CSV + MV \cdot MTT \cdot ECMV + \\ &SDC \cdot WC \cdot v \cdot CO_o \cdot DP \end{aligned} \quad (16)$$

The first term of Eq. (16) corresponds to the response (system) costs (associated with the dispatched equipment units) minus the response benefit resulting from the (partial or total) recovery of the spilled oil. The second and third terms correspond to the response (system) costs associated with the supply vessels and the mother vessel, respectively. The last term corresponds to the non-response costs.

Note that the response costs and the response benefit are optimised as the non-response costs are determined from the data of the problem. Hence, the values of the decision variables do not influence the non-response costs.

The response costs should be less than the response benefit (i.e. their algebraic sum should take a negative value, which signifies a benefit in the cost/benefit weighted balance) for the dispatch of response equipment to be justifiable. Moreover, the absolute value of the algebraic sum of the response costs and the response benefit should not exceed the non-response costs; otherwise, excessive expenses will be devoted to the clean-up of the spill.

4.3 Constraints

The decision variables are allowed to take values that satisfy the following constraints:

Constraint 0: The decision variables that determine the number of response units that will be dispatched from each facility to the spill site are non-negative integers (this integrality is relaxed). The decision variables that determine whether a mother vessel will be dispatched to the spill site and whether a supply vessel will be dispatched from a facility are binary (0/1) variables.

$$\begin{aligned} &a)\ x_{ie} \geq 0 \ \text{ integers} \ \forall\, i,\, e \\ &b)\ SV_i,\, MV \in \{0,1\} \ \forall\, i \end{aligned} \quad (17)$$

Integrality in constraint (a) is relaxed and the relaxed IP problem is the one to be solved (actually an MIP problem). In practice, the solution is integral or near-integral (see Iakovou et al., 1996) and in the latter case, it is rounded to obtain a near-optimal solution.

Constraint 1: The number of units of equipment type e dispatched from facility *i* to the spill site cannot exceed the total number of units of type *e* stored at *i*.

$$x_{ie} \leq N_{ie} \quad \forall\, i,\, e \quad (18)$$

Constraint 2: The total response clean-up capacity (in tonnes / h) adjusted with the operational efficiency rate of each equipment type that is dispatched to the spill site and operates for the available clean-up time must be limited. A limit can be set by the spill size (tonnes) multiplied by the desired coverage coefficient CC. Also, the weathering coefficient is taken into account (see Constraint 8). Note that a CC greater than 1 signifies the dispatch of extra capacity to compensate for operational clean-up inefficiencies and introduces a safety margin.

$$\sum_i \sum_e x_{ie} \cdot RE_e \cdot u_e \cdot T_{ie} \leq CC \cdot WC \cdot v \quad (19)$$

Constraint 3: The dispatch of equipment of type e to the spill incident is possible only when type e is operational with respect to the oil type, the weather conditions and the sea type at the spill site. A dummy variable is used for this purpose.

$$\sum_i x_{ie} \leq DC_e \ \forall\ e \quad (20)$$

Constraint 4: The dispatch of equipment of certain type to the spill theatre should be made only when the time limitations are respected. The time limitations require that a response unit reaches the spill before the spill hits the shore (this limitation can be relaxed) and that the available time for its clean-up operation (that is the time from the moment the unit arrives at the spill until the spill hits the shore) is greater than a minimum time period set by the decision-maker. A dummy variable is used for this purpose.

$$x_{ie} \leq DT_{ie} \ \forall\, i,\, e \quad (21)$$

Constraints 5: The following constraints require that a supply vessel is to be used whenever the dispatch of certain response equipment (e.g. *e* = 1...m) from a facility to the spill site is to take place.

$$\begin{aligned} &a)\ SV_i \cdot L1 \geq \sum_{e=1}^{m} x_{ie} \quad \forall\ i \\ &b)\ SV_i \leq \sum_{e=1}^{m} x_{ie} \quad \forall\ i \end{aligned} \quad (22)$$

$L1$ that appears in Eq. 22(a) can be any integer greater than the sum of all available units of types *e* = 1...m

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[1] The fixed costs of opening some or all of the candidate response facilities and the variable acquisition and storage costs of the response equipment are sunk costs in the tactical decision-making problem. They are addressed in the strategic decision-making problem.

stored at the response facilities.

Constraints 6: The following constraints require that a mother vessel is to be used whenever the dispatch of certain response equipment (e.g. $e = \{1,2,3\}$) (and therefore of at least one supply vessel) from a facility to the spill site is to take place (if the supply vessel to be dispatched is only one, that will also be the mother vessel). Constraints 5 and 6 basically refer to the EU-MOP response concept.

$$\begin{aligned} &a)\ MV \cdot L2 \geq \sum_i SV_i \\ &b)\ MV \leq \sum_i SV_i \end{aligned} \tag{23}$$

$L2$ that appears in Eq. 23(a) can be any integer greater than the sum of all response facilities (or possibly dispatched supply vessels).

Constraint 7: The absolute value of the algebraic sum of the response costs and the response benefit should not exceed the non-response costs, i.e. the total cost is positive or zero.

$$Total\,Cost \geq 0 \tag{24}$$

Constraint 8: The weathering effects on the size of the spill are taken into account. Specifically, relevant weathering coefficients (that determine the remaining spill size) are calculated for the points in time when the response equipment arrives at the spill site from the different facilities that actually dispatch equipment[1]. The maximum of these coefficients is taken into account to determine the amount of oil that is left available for collection by the response equipment (assumption). This coefficient is also applied for the calculation of the spill's potential damage.

The weathering coefficient WC is the maximum of the weathering coefficients corresponding to the remaining spill size (because of weathering effects only) when the response equipment from each facility i arrives at the spill site (assumption).

$$WC = \max(WC_i) \tag{25}$$

Constraint 9: The viscosity of the spilled oil when the response equipment from facility i arrives at the theatre of the incident should be within the operational (technical) limits of the equipment. Otherwise, the respective equipment will not be dispatched from facility i. A dummy variable is used for this purpose.

$$x_{ie} \leq DL_{ie}\ \forall i, e \tag{26}$$

5. Model specifics

The input data required for the model to run can be distinguished in spill data, the cost coefficient, response equipment data and facilities data.

5.1 Spill data

The required spill incident data include:

- spill size (in tonnes);
- spill oil type;
- fresh oil properties (density, viscosity)
- temperature and wind profiles
- oil type characteristics (persistent / non-persistent and cost in euros / tonne);
- weathering effects on the spill size, i.e. the remaining size of the spill as a function of time and as a result of weathering only (input from the spill fate model);
- viscosity of the spilled oil as a function of time (input from the spill fate model);
- spill coverage coefficient;
- sea type (e.g. open ocean area, enclosed sea, shallow water area);
- weather conditions category (calm / moderate / rough);
- spill direction ("moving towards shore" or "away from shore");
- distance from shore (in nautical miles);
- speed approaching shore (knots);
- additional time for response (hours) – relaxation of preset time limit (i.e. the time it would take the spill to hit the shore).

Note that apart from the initial spill size, the fresh oil properties, the wind and temperature profiles, the rest of the input parameters for the oil fate model are set as described in section 3.

The spill incident is placed in a damage potential group (A - High, B - Medium, C - Low), according to an internal calculation routine. This routine is extensively used in the strategic model, where numerous spill sites and potential spill incidents are examined. In the tactical problem we need to place a known spill incident in a damage potential group. The required input for the calculation routine and for the spill site, where the incident occurs, should therefore be available to the decision maker from a database, constructed before the tactical decision-making point is reached. The required input is:

- a damage potential coefficient assigned to each group A, B, C;
- a score range (scale of 1-10) set for placing a spill incident in each group;
- a score 1-10 (from least sensitive to most sensitive) depending on the spill site geographical location;
- a relative weight for each of the target categories: "fish", "birds", "mariculture" and "beach-tourism";
- a score 1-10 (from least to most severe) regarding the relative impact of "size", "oil type", "distance from shore", "direction of spill", and "location" on potential damage to "fish", "birds", "mariculture" and "beach-tourism";
- a score (scale of 1-10) for the possible values of the parameters "size", "oil type", "distance from shore", and "direction" of a spill incident.

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[1] For the calculation of any of these coefficients, it is assumed that the spill size has changed up to that point in time due to weathering only, and not due to possible collection of oil from previously arrived response equipment.

5.2 The System/damage costs coefficient

The system/damage costs coefficient is set by the decision-maker. This coefficient expresses the relative value of 1 monetary unit spent for the response system compared to 1 monetary unit of spill damage. It represents how much the decision maker is willing to pay in system costs in order to reduce damage costs by 1 monetary unit (e.g. 1 euro).

Setting to this coefficient a value greater than 1 means that greater value is placed on the spill damage cost, rather than to the system cost (compared to what the monetary values of these costs suggest). Or in other words, a high value of the coefficient increases the relative importance of damage costs vis-à-vis system costs. A default value would be 1 (i.e. the relative monetary values represent the actual relative weight placed on the system / damage costs).

5.3 Response equipment data

The required data for each response equipment type include:

- nominal oil recovery capacity (tonnes/h);
- recovery efficiency rate;
- operational/clean-up cost (euros/h);
- weather conditions operational limits (calm/moderate/rough);
- type of oil operational limits (e.g. persistent /non-persistent);
- type of sea operational limits (e.g. open ocean area, enclosed sea, shallow water area);
- clean-up operations minimum time length (h);
- supply vessel cost/rental rate (euros/h);
- extra cost for mother vessel/rental rate (euros/h).

5.4 Response facilities data

The required data for each response facility include:

- distance (nautical miles) from the spill site;
- number of units stored from each equipment type;
- average response speed (knots) for each equipment type;
- transportation cost (euros/mile) per unit of each equipment type.

6. Case study

6.1 Case overview

In this section, the previously presented model-based approach for tactical decision making in oil spill response is applied in the case of a real spill incident.

On 29 August 2000, the bulk carrier Nordland grounded in strong winds off Kythira Island, Greece. An estimated 110 tonnes of IFO180 was spilled, which drifted towards the shore of a nearby village, contaminating a few kilometres of coastline and a small fishing harbour. The incident data have been provided by EPE (2006). The variation in the dominant environmental conditions, namely wind speed and temperature, are shown in Figures 6 and 7, respectively. The IFO 180 intermediate fuel oil belongs to class group 4 and has an API degree of 14.7^{o}. Moreover, its density is $0.9670\ kg/m^3$ at $15^{o}C$, its dynamic viscosity 2324 cP at $15^{o}C$ and the pour point $-10\ ^{o}C$. In the actual Nordland accident, it has been reported that no significant emulsion was found (EPE, 2006); therefore, in our calculations we have excluded the emulsification process. Moreover, in our computations we have incorporated the actual environmental conditions at the spill scene.

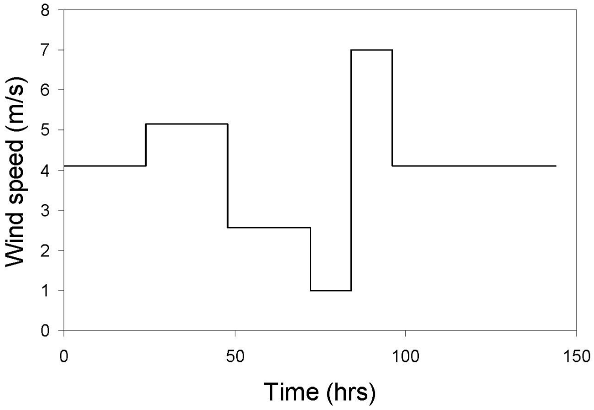

Fig. 6: Wind speed variation.

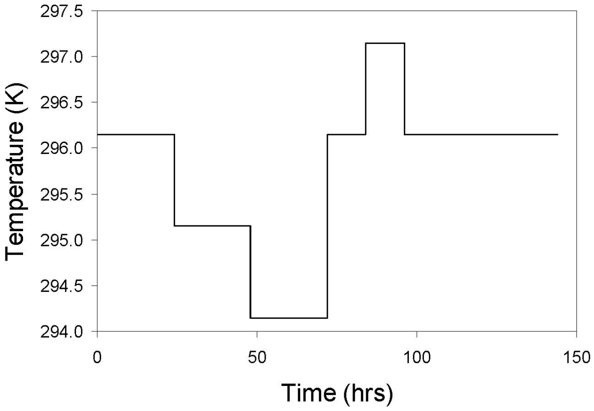

Fig. 7: Temperature variation.

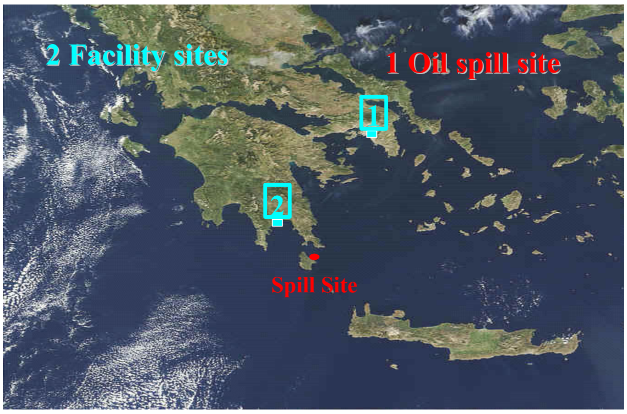

Fig. 8: Tactical response setting.

The spill site and the response facilities are shown in Figure 8. The 2 response facilities are listed in Table 1 with their distances from the spill site.

Table 1: Response Facilities

Facility	Port	Distance from spill site
1:	Piraeus	120 NM
2:	Githion	45 NM

Indicatively, certain model data are provided below.

Spill data:

- Type of sea: Sea Type II (enclosed area);
- Weather conditions: moderate;
- Spill coverage coefficient: 1;
- Spill direction: towards shore;
- Distance of incident from shore: 0.5 NM;
- Speed approaching shore: 0.1 kts;
- Damage potential group: A (output of damage assessment routine according to preset criteria).
- System / damage cost coefficient: 1

Response equipment data: From the 4 different EU-MOP response equipment types, only types 2 and 4 are considered in the simulation, which are allocated to the response facilities as shown in Table 2[1].

Table 2: EU-MOP Response Equipment Types: units' allocation per facility

Facility:	**(1) Piraeus**	**(2) Githion**
Type 1: Small Size	-	-
Type 2: Medium Size - Catamaran	-	8
Type 3: Large Size - Monocat	-	-
Type 4: Large Size - Catamaran	2	-

The basic characteristics of all 4 EU-MOP response equipment types are given in Table 3.

Table 3: Response Equipment Data – set 1

Equipment type=	**1**	**2**	**3**	**4**
Oil recovery capacity (tonnes/h)	0.30	**1.35**	2.50	**2.50**
Recovery efficiency rate	0.66	**0.66**	0.66	**0.66**
Operational / clean-up cost (euros/h)	20	**150**	200	**200**
Weather conditions operational limits (1:operational; 0: not operational)				
calm	1	**1**	1	**1**
moderate	0	**1**	1	**1**
rough	0	**0**	1	**1**
Type of sea operational limits (1:operational; 0: not operational)				
sea-type I	0	**0**	1	**1**
sea-type II	0	**1**	1	**1**
sea-type III	1	**1**	0	**0**

The data shown in Table 4 were also used for the response from each facility.

Table 4: Response Equipment Data – set 2

Equipment type:	**2**	**4**
Average response speed (kts)	8	8
Transportation cost (euros/mile)	15	20

Also:

- Supply vessel & other related daily costs: 5,170 euros/24h
- Mother vessel & other related daily costs: 16,140 euros/24h

The time limitations, that have been set, define the total time length for operations to 25 hours and also require that any unit will be dispatched to the spill theatre if it is to operate for a minimum of 6 hours.

6.2 Simulation of the oil spill fate

The mathematical model presented in section 3 is used to simulate the weathering behaviour of the spill. The model is implemented in the gPROMS process modelling tool (Process Systems Enterprise Ltd, London, UK). We consider the spillage of 110 tones of oil in a continuous fashion assuming a discharge rate of 18 m^3/hr. This is feasible with the implementation of the model that we have followed. The oil fate model is run from the beginning of the spill, so that for every amount of oil present at sea, weathering is applied. In the implementation of the model we have incorporated the step changes in the wind speed and temperature conditions shown in Figures 6 and 7.

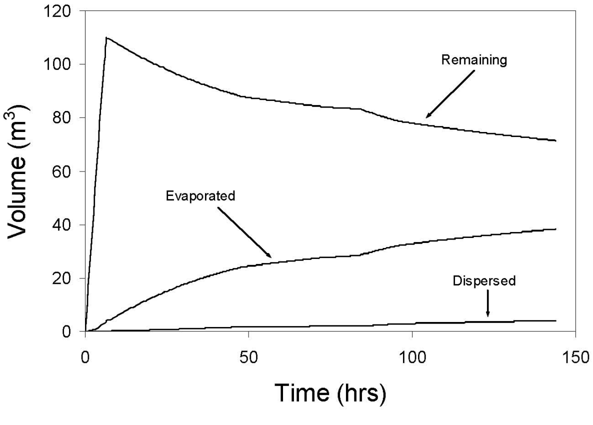

Fig. 9: Spill volume dynamics as affected by evaporation and dispersion.

The dynamic model is simulated for 145 hours and in Figure 9 we present the dynamics of the spill volume as affected by evaporation and dispersion. As the weathering and spreading processes evolve, there is a gradual reduction in the total fluid volume, mainly driven by evaporation. Figure 10 shows the effect of spreading on the slick area and the decrease of its thickness as a result of both weathering and spreading. In practice, after the first 65 hrs from the spillage, the oil spreads over more than 0.17 km^2 having become a thin film of less than 0.5 mm. Figure 11 presents model predictions for the dy-

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[1] The selection of these two equipment types to be allocated at the respective response facilities is the outcome of a strategic level simulation (not presented in this paper).

namics of indicative physical properties like viscosity and density. The increase shown is attributed to the effects of evaporation. As the IFO 180 oil is considered to not emulsify, the variation of physical properties is not very pronounced. Also, note the way that all trajectories –and in particular viscosity- are affected by the introduction of the wind and temperature step changes (see the kinks in Figures 9 and 11). This is a more realistic approach of the dynamic effects that the environmental conditions have on the oil spill fate.

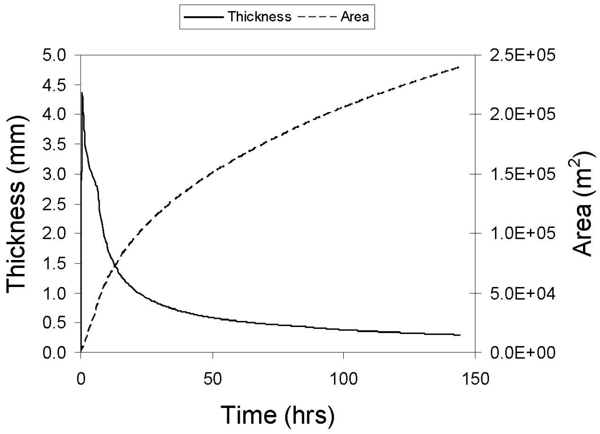

Fig. 10: Slick thickness and area predictions.

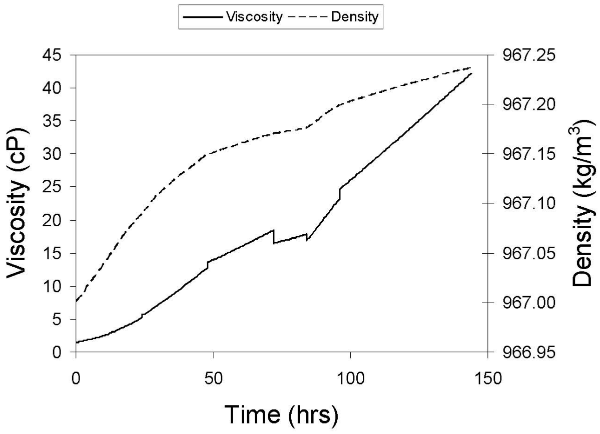

Fig. 11: Model predictions for the dynamics of viscosity and density.

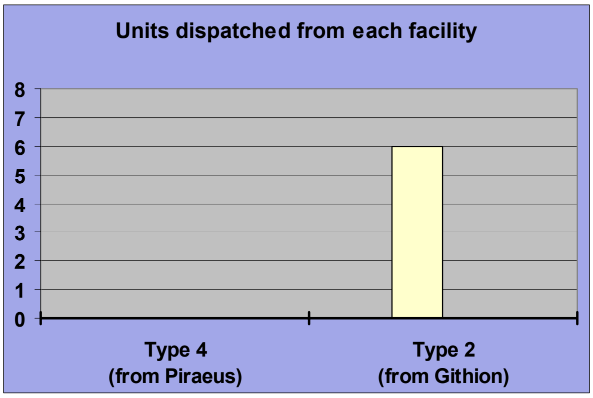

Fig. 12: Units dispatched (Run 1).

6.3 Tactical response results

Given the above data the tactical decision-making model provides the results shown in Figure 12 (Run 1). Hence, 6 "Type 2" (Medium Size - Catamaran) EU-MOP units are dispatched to the spill site from Facility 2 (Githion). They arrive at the spill theatre in 5.6 hours and collect all 93 tonnes of oil (available for collection at that time at the incident theatre, weathering effects taken into account) within 25 hours (which was the preset time limit for the operations).

If the total time of operations is reduced to 15 hours (Run 2), then the results are shown in Figure 13: all 8 "Type 2" (Medium Size - Catamaran) EU-MOP units are dispatched to the spill site from Facility 2 (Githion). They arrive to the spill theatre in 5.6 hours and collect 67 tonnes of the 93 tonnes of oil (available for collection) within the 15 hours. No units are dispatched from Piraeus, because it would take 15 hours to arrive at the spill theatre and there would be no available time for operations (the minimum has been set to 6 hours).

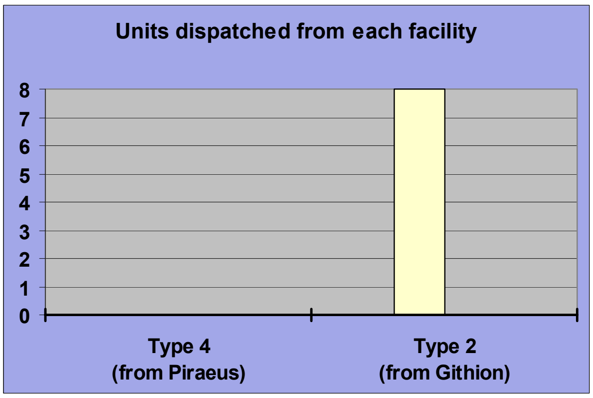

Fig. 13: Units dispatched (Run 2).

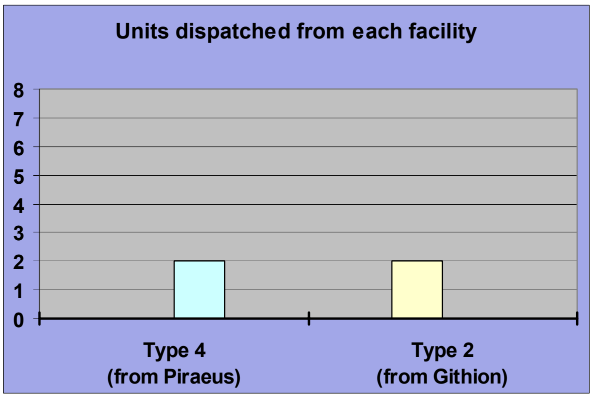

Fig. 14: Units dispatched (Run 3).

On the other hand, if the total time of operations is increased, for instance, to 35 hours as in Run 3, then the results are to send 2 "Type 4" (Large Size - Catamaran) units from Facility 1 (Piraeus) and 2 "Type 2" (Medium Size - Catamaran) units from Facility 2 (Githion) (see Figure 14). All 93 tonnes of oil available for collection are collected within the 35 hours. In this case, the dispatch of large units from Piraeus is possible, because the respective time limitations are no longer violated.

In a similar manner, sensitivity analyses can be performed for other parameters. For example, with a decrease in the average response speed from 8 kts to 4 kts (notification and mobilisation delays of the response are included in this average response speed) and the rest of the data as in Run 1, the results change (Run 4). In this case, all 8 “Type 2” (Medium Size - Catamaran) EU-MOP units are dispatched to the spill site from Facility 2 (Githion) as shown in Figure 15. They arrive at the spill theatre in 11.25 hours (rather than 5.6 hours in Run 1) and within the 25 hours of total operations they are able to collect the 98 of the 103 tonnes of oil available for collection following the weathering, the effects of which are more evident in this case and also result in the escape of some amount of oil.

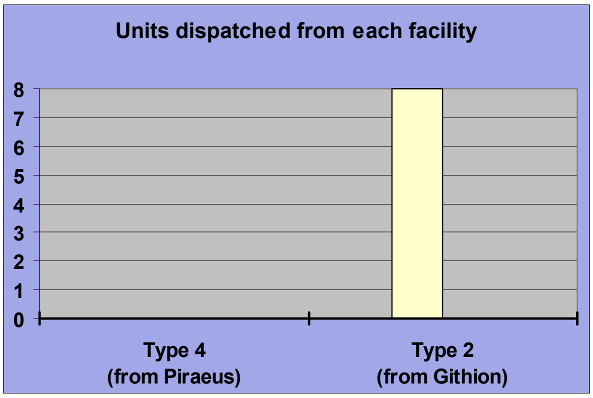

Fig. 15: Units dispatched (Run 4).

As another example, an increase in the operational efficiency rate of “Type 2” units from 66% to e.g. 80% and the rest of the data as in Run 1, gives the results shown in Figure 16 (Run 5): 5 “Type 2” (Medium Size - Catamaran) EU-MOP units can do the same job that 6 units did in Run 1.

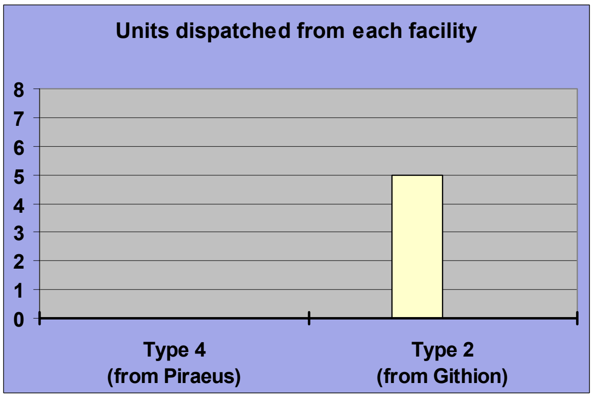

Fig. 16: Units dispatched (Run 5).

Instead, a decrease in the operational efficiency rate of “Type 2” units to 40%, and the rest of the data as in Run 1, gives the results shown in Figure 17 (Run 6), i.e. all available units are dispatched and they collect all 93 tonnes of oil within 25 hours. A further decrease in the operational efficiency rate of “Type 2” units to 25% (Run 7) also results in the dispatch of all available units (as in Figure 17); however, they manage to recover 85 out of the 93 tonnes available for collection within the 25 hours of operations.

Figure 18 shows for the previous Runs 1 to 7 the deviation of the results from the theoretically optimal solution. The latter corresponds to an achieved Total Cost (optimised response cost + non-response cost) equal to 0. In Runs 2, 4 and 7, this deviation is high due to the fact that only a fraction of the spilled oil was removed in these cases. The deviation in the other Runs is up to 2%. This figure is practically an index of the ability of the model to reach the optimal solution under the specific assumptions/constraints: the Total Cost is constrained to take a non-negative value, while the zero value corresponds to the ideal response, where all oil is collected and the potential damage of the spill is totally eliminated.

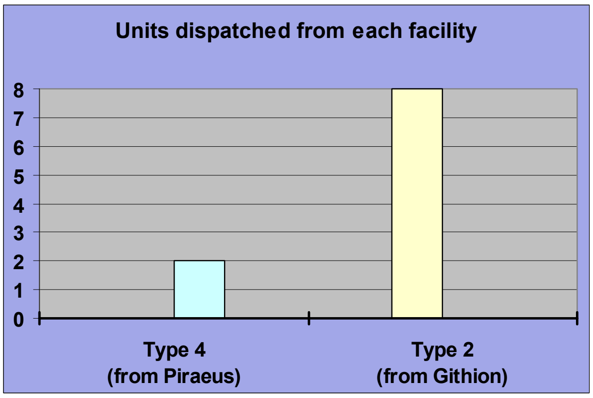

Fig. 17: Units dispatched (Run 6 & 7).

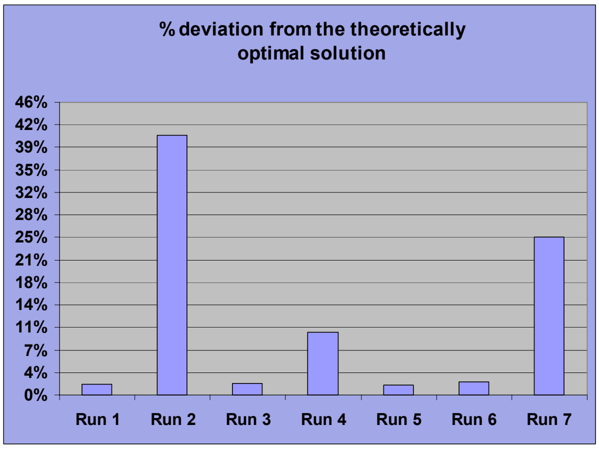

Fig. 18: Optimality of solutions (Runs 1 - 7)

According to the model results one mother vessel is used, which also serves as the supply vessel for the transport of the equipment to the spill site. The operational costs associated with the vessel are about €24,000 for the whole operation and the costs associated with the operation of the EU-MOP units *per se* are about €4,300. All the oil available for collection was removed and practically the potential (non-response) damage cost estimated to €2,860,000 was eliminated.

The damage-related cost estimations obviously depend on the model inputs. For example, an increase of the value of the system/damage cost coefficient from 1 to 2 (which places double value on 1 cost unit of potential damage from the spill, compared to 1 cost unit spent on response), did not change the number of units dispatched to the spill site (this was expected, all the spilled oil had been collected in the first case anyway), nor the response costs. However, the figures of the damage cost estimations doubled accordingly.

7. Conclusions

In this paper, we have presented a model that supports the oil spill response decision-making process at the tactical level, to determine the actions required to respond to a specific spill that has already occurred. This problem determines from which facilities to dispatch response units to the spill site and, moreover, the types and quantities of the units to be dispatched.

In this work we have proposed an integrated approach, in which the tactical optimization problem is sequentially coupled with a dynamic mathematical model that provides estimates of the oil spill fate at the contact time of the spill with the response means. The model consists of a set of differential and algebraic equations that describe the dynamics of an oil spill as these are affected by spreading and weathering. To describe the tactical problem, an integer optimization problem is then formulated where the objective is to minimize a total cost function by balancing the response system costs and the resulting reduction in the spill damage costs.

Besides the model integration, the tactical model is novel in many respects. It explicitly takes into account a number of cost and technical parameters and introduces a number of realistic operational constraints. Yet, it is simple enough to operate as a stand-alone model. The optimization program is solved by applying the mixed-integer programming theory. Also, a user-“calibrated” built-in routine is utilised to assess the damage potential of any spill incident. Moreover, a complicated response infrastructure is supported that requires the dispatch of a mother vessel and possibly more than one supply vessels for the transportation of response units from the facilities to a spill site, in accordance with the EU-MOP concept.

An illustrative application of the model was presented to demonstrate its modelling potential in solving complex tactical decision-making problems. The model also allows performing sensitivity analyses with respect to its input data and assumptions easily.

Our approach is to respond optimally on a cost/benefit basis. Although the response time is not optimised, time constraints are applied (e.g. response before the spill hits the shore). As a suggestion for further research, another approach could set as primary objective the minimisation of the response time or the maximisation of the coverage of the spill (e.g. as in Belardo et al, 1984). In addition, the amount of oil that is captured on the shore, once the spill arrives there, and the re-entrainment of the rest back to the sea are parameters to be addressed in a future enhanced version of the model (that is already under preparation). It should also be noted that the above model is part of a work that also addresses the strategic level decision-making, to be presented in another publication.

8. Acknowledgements

Our work was supported by the European Commission within the 6th Framework Programme, Priority 1.6.2, Sustainable Development, Global Change and Ecosystems, under contract number FP6-2003-516221 (EU-MOP: Elimination Units for Marine Oil Pollution), for which we are grateful.

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